A Theory of Exchange Rates and The Term Structure of Interest Rates†

HYOUNG-SEOK LIM* and MASAO OGAKI*

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Korea Institute of Finance

Keio University

ABSTRACT

This paper defines the concepts of indirect and direct risk premium effects and analyzes their properties in an exchange rate model. In the model, these effects are endogenously determined in a rational expectation equilibrium. For the effect of an interest rate shock in the domestic country on the exchange rate in the model, the direct and indirect risk premium effects have the opposite signs, and the indirect risk premium effect works in a counter-intuitive way. Under reasonable parameters, the indirect risk premium effect can dominate the direct risk premium effect. This means that the domestic short-term bonds and foreign bonds are complements in the model even though domestic long-term bonds and foreign bonds are substitutes. We also show that the indirect risk premium effect can be a factor in explaining stylized facts regarding Uncovered Interest Parity for short and long-term interest rates. In particular, the forward premium anomaly is found for short-term interest rates, but not for long-term interest rates. Our model, which focuses on the indirect risk premium effect and on the term structure of interest rates, can be combined with a small sample bias approach to explain stylized facts about the forward premium anomaly.

Keywords: forward premium anomaly, Uncovered Interest Parity for long-term bonds

JEL Classification: F31, G11

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* Korea Institute of Finance, Seoul, Korea. E-mail: hslim@kif.re.kr.

☆ Corresponding author, Department of Economics, Keio University, 2-15-45 Mita, Tokyo 108-8345, E-mail: mogaki@econ.keio.ac.jp
1 Introduction

Uncovered Interest Parity (UIP) states that the interest rate return on a domes-
tic currency asset should equal the interest rate on each foreign currency asset, less
the expected appreciation of the domestic currency. However, contrary to what UIP
would predict, many empirical findings about short-term interest rates have shown
that currencies with high interest rates tend to appreciate. This is called the forward
premium anomaly. On the other hand, more favorable evidence for UIP has been
found for long-term interest rates.\footnote{See the literature review in Section 2.}

This paper defines the concept of an indirect risk premium effect and analyzes
properties of this effect in an exchange rate model. To investigate the role of the indi-
rect risk premium effect in explaining the stylized facts mentioned above, we build a
partial equilibrium model of exchange rate determination for a small open economy.
The domestic investors have a constant absolute risk aversion utility function over
their wealth in the next period. The asset returns are normally distributed condi-
tionally on the available information. We assume that there are three assets in the
model: one risk-free asset - domestic short-term bonds; and, two risky assets - do-
mestic long-term bonds, and foreign bonds. The investors are also assumed to have
a short investment horizon in our model. Employers of professional traders active
in foreign exchange markets are likely to assess traders based on their short-horizon
performances. Therefore, our short investment horizon assumption is justifiable.

Given the conditional expectations and variances of all risky assets, we can
decompose the effect of a change in the domestic short-term interest rate on the
demand for foreign bonds into two components. First is the \textit{direct risk premium}
effect. It is the change in demand due to changes in the risk premium for foreign bonds when the risk premium for domestic long-term bonds is kept constant. The other is the indirect risk premium effect. It is the change in demand due to changes in the risk premium for domestic long-term bonds when the risk premium for foreign bonds is kept constant. The change in the demand for foreign bonds is the sum of these direct and indirect risk premium effects. In the special case of risk-neutral investors, the indirect risk premium effect does not play any role. However, when investors are risk-averse, it is necessary to evaluate both the direct and indirect risk premium effects in order to study how the foreign exchange rate changes when the domestic short-term interest rate changes.

The direct and indirect risk premium effects are properties of the demand for foreign bonds, given the distributions of the asset returns and wealth, which are conditional on the information available to the investors. In order to examine how these effects work in equilibrium, we consider the term structure of interest rates, where some shocks to short-term interest rates are not transmitted to long-term interest rates, even though they affect the risk premium of long term bonds.

Let us suppose that the domestic short-term interest rates rise but the long-term interest rates do not change. First, the risk premium for foreign bonds falls and the direct risk premium effect lowers the demand for foreign bonds without changing the exchange rate. Since the supply for foreign bonds is essentially fixed in the short-run by the cumulative current balance in the model, the domestic currency appreciates, creating expected future depreciation of the currency in order to restore equilibrium. Second, the risk premium for domestic long-term bonds also falls as the domestic

\textsuperscript{2}A more general definition of the indirect risk premium effect is given in Section 3.
short-term interest rates rise. If the conditional covariance of the two risky assets’ returns is positive, the indirect risk premium effect increases the demand for foreign bonds. In order to restore equilibrium, the domestic currency must depreciate this period, creating an expected appreciation in response to the indirect risk premium effect.

The sign and magnitude of the indirect risk premium effect depends on the conditional covariance. In the partial equilibrium model with given stochastic processes for interest rates, we endogenously derive the demand for foreign bonds by solving for the rational expectation equilibrium of the conditional expectation, variance, and covariance of the exchange rate. In equilibrium, the conditional covariance of the two risky assets’ returns is positive, and the direct and indirect risk premium effects have opposite signs. We show that under some reasonable parameter configurations, the indirect risk premium effect dominates the direct risk premium effect, even when the degree of risk aversion is low. As a result, the domestic currency depreciates when the domestic short-term interest rates rise and the long-term interest rates do not rise. This counter-intuitive result helps in making the model more consistent with the stylized facts regarding UIP.

On the other hand, when domestic long-term interest rates also rise with domestic short-term interest rates, the risk premium for domestic long-term bonds does not change. In this case, the indirect risk premium effect does not affect the equilibrium exchange rate. Therefore, the domestic currency appreciates when the short-term and long-term interest rates rise together. In this way, our model is consistent with stylized facts about exchange rates and the long-term interest rates. The intuition behind these results for direct and indirect risk premium effects can be generalized
with Ogaki’s (1990) concepts of direct and indirect substitution effects to the cases of other utility functions and more than three assets as explained in Section 4.

In this paper, we show that the indirect risk premium effect is likely to be quantitatively more important than the direct risk premium effect. In particular, we show that the indirect risk premium effect can even dominate the direct risk premium effect under reasonable parameter configurations. Our model still predicts that the slope coefficient in the forward premium regression for short-term interest rate differential converges in probability to a positive number even when the indirect risk premium effect dominates. However, the dominant indirect risk premium effect is likely to cause downward small sample bias when the exchange rate is persistent. A Monte Carlo simulation for short-term regressions based on this parameter specification consistently shows the forward premium anomaly. The stronger the indirect risk premium effect, the more statistically significant the negative slope coefficient.

The result in this paper is in sharp contrast to the conventional view that short-term capital is more internationally mobile than long-term capital. Operation Twist in the 1960, in which the Federal Reserve and the Treasury attempted to raise the short-term rate relative to the long-term interest rate, was evidently based on this view. However, empirical work by Fukao and Okubo (1984) suggests that international factors are more important in determining the domestic long-term interest rates than in determining short-term rates. Popper (1993) presents empirical evidence that long-term capital is as internationally mobile as short-term capital.

The model in this paper has policy implications. It implies that the effectiveness of central bank attempts to affect exchange rates through the control of short-term
interest rates depends on the responsiveness of long-term interest rates to changes in short-term interest rates.

The rest of the paper is organized as follows. Section 2 provides a literature review, while section 3 defines the direct and indirect risk premium effects. Section 4 presents the model and Section 5 derives the rational expectation equilibrium. First, given the covariance and variance assumed by agents, the rational expectation of the mean of the exchange rate is used to solve for the exchange rate’s law of motion. Then the condition for the rational expectation for the covariance is derived. Finally, the unique stable rational expectation equilibrium is found by equating the variance assumed by agents with the one implied by the demand function. Section 6 investigates the model’s implications on the relationship between the exchange rate and the term structure of interest rates, then concludes with directions for future work.

2 Literature Review

For short-term interest rates and forward exchange rates, UIP is typically rejected (see e.g., Hodrick (1987), Engel (1996), and Sarno (2005) for recent surveys). As Fama (1984) finds that one reason many papers reject UIP is that the regression of future depreciation on the current forward premium (which is equal to the short-term interest rate differential under the covered interest parity) yields negative estimates of the slope coefficient. This is called the forward premium anomaly (see Backus, Foresi, and Telmer (2001) for a detailed discussion).

For long-term interest rates, more favorable evidence for UIP has been found (see e.g., Chinn (2006) for a recent survey). Direct evidence is given by recent papers,
such as Alexius (2001) and Chinn and Meredith (2005). They find that regressions of future depreciation over a long horizon on the current long-term interest rate differential typically yield significantly positive estimates of the slope coefficient.\footnote{Alexius (2000) finds similar results for returns on long-term bonds over short investment horizons.} Indirect evidence has been found in the standard exchange rate models, such as Meese and Rogoff (1988), Edison and Pauls (1993) and Baxter (1994). They show that the long-term interest rate differential is more consistent with the assumptions of UIP and long-term PPP than the short-term rate differential. Similarly, implications of standard exchange rate models hold better in long-horizon data than in short-horizon data (see e.g., Mark (1995)).

It is challenging to find an economic explanation for the forward premium anomaly for short-term interest rates. Neither the standard consumption-based asset pricing model with risk-averse investors (see e.g., Mark and Wu (1998)) nor the dynamic term structure model (see e.g., Wu (2005)) can explain it. Several explanations have been provided to meet this challenge. Alvarez, Atkeson, and Kehoe (2002, 2009) construct a model of segmented asset markets which can be consistent with the forward premium anomaly. McCallum (1994) and Chinn and Meredith (2004) provide an explanation for the forward premium anomaly based on policy reactions. However, in their models, an unspecified error term is necessary for the uncovered interest parity relationship. Bacchetta and van Wincoop (2009) show that infrequent foreign currency portfolio decisions can explain the anomaly. Fisher (2006) explains the forward premium anomaly in terms of a model where agents have diverse prior beliefs about domestic and foreign inflation. If some agents have diffused priors about
a country’s inflation process, then its one-month forward rate will be negatively correlated with realized depreciations. Except for Chinn and Meredith (2004), these explanations are focused on the anomaly for short-term interest rates and do not show formal analysis of whether or not their explanations are also consistent with more favorable evidence for UIP for long-term interest rates.

Another way to explain the forward premium anomaly emphasizes small sample econometric problems. Baillie and Bollerslev (2000), for example, argue that the forward premium anomaly can be viewed mainly as a statistical phenomenon caused by small sample sizes and persistent autocorrelation in the forward premium (also see e.g., Maynard and Phillips (2001) and Maynard (2003)).

In this paper, we show that the indirect risk premium effect alone does not solve the anomaly for short-term interest rates. However, the effect can be complementary to a solution for the anomaly in building a model that is consistent with both the anomaly and more favorable evidence for UIP for long-term interest rates. We also show that our model complements the explanation that emphasizes the small sample econometric issue. We observe these patterns with high probability in small samples. However, the indirect risk premium effect could be also introduced to any economic model that solves the anomaly for short-term interest rates.

3 Indirect Risk Premium Effect

This section defines the concept of the indirect risk premium effect. Consider an economic agent who considers allocating his wealth between \( N \) different assets in period \( t \). Given the distributions of all returns in the future and the variance, covariance, and higher moments of returns from period \( t \) to period \( t + 1 \) conditional
on the information set available period $t$, we can write the demand function for an asset $i$ as $A_{i,t}(\rho_{1,t}, \ldots, \rho_{i,t}, \ldots, \rho_{N,t}, r_t, W_t)$, where $\rho_{j,t}$ is the risk premium for asset $j (j = 1, \ldots, N)$, $r_t$ is the risk-free short-term interest rates, and $W_t$ is the wealth. In this paper, we are interested in the effect of a change in $r_t$ on the demand, given the expected returns of the assets. Because a rise in $r_t$ decreases each risk premium, the effect is given by

$$\frac{\partial A_{i,t}}{\partial \rho_{i,t}} - \sum_{j \neq i} \frac{\partial A_{j,t}}{\partial \rho_{j,t}} + \frac{\partial A_{i,t}}{\partial r_t}$$  \hspace{1cm} (3.1)$$

If the agent is risk neutral, then the first term is infinite and dominates the other two terms. We define the direct risk premium effect to be this term,

$$-\frac{\partial A_{i,t}}{\partial \rho_{i,t}}$$  \hspace{1cm} (3.2)$$

and the indirect risk premium effect$^4$, respectively.

$$-\sum_{j \neq i} \frac{\partial A_{j,t}}{\partial \rho_{j,t}}$$  \hspace{1cm} (3.3)$$

Our intuition tends to focus on the direct risk premium effect, but we show that the indirect risk premium effect can dominate the direct risk premium effect, even when the agent’s risk aversion is fairly small. In our model, we assume that the agent’s absolute risk aversion is constant, and the returns are normally distributed, conditional on the information set. Under these assumptions, the third term in 3.1 is zero.

$^4$The concepts of direct and indirect risk premium effects are closely related, but differ from direct and indirect substitution effects defined by Ogaki (1990).
4 The Model

This paper adopts a simple partial equilibrium exchange rate model following Driskill and McCafferty (1980) and Fukao (1983). We endogenously derive the demand for foreign bonds by solving for the rational expectation of the covariance, so that the covariance assumed by agents is consistent with the one implied by the demand function.\(^5\) It is technically difficult to solve for the rational expectation of the covariance in complicated asset pricing models. For this reason, we employ three asset models.

Consider a partial equilibrium model of exchange rate determination. For simplicity, the overall price level is assumed to be constant. Alternatively, all variables can be considered to be measured in real terms. Investors live for two periods, and the same number of investors is born every period. There are three assets: domestic short-term bonds, \(B_{S,t}\), domestic long-term bonds, \(B_{L,t}\), and, foreign bonds, \(B_{F,t}\). Since the foreign interest rate is assumed to be constant, the foreign short- and long-term bonds are perfect substitutes and do not need to be distinguished. The domestic short- and long-term bonds are discount bonds paying one unit of the domestic currency after one period and two periods, respectively. The foreign bonds behave in the same manner. At time \(t\), a representative investor allocates his initial wealth \(W_t\) among the three assets and he collects the payoffs paid by the assets he holds at the beginning of time \(t + 1\).

Let \(q_t\) be the price of domestic long-term bonds at time \(t\). Let \(r_t\) be the domestic short-term interest rate, and let \(R_t\) be the domestic long-term interest rate. Then,

\(^5\)The demand function depends on the covariance, conditional on the available information, between the exchange rates and the short-term interest rates. At the same time, the demand for foreign bonds affects the dynamics of the exchange rate and the covariance.
the rate of return on holding domestic long-term bonds for one period, $r_{L,t}$, is:

$$
(4.1) \quad r_{L,t} = \frac{1}{q_t} \left( \frac{1}{1 + r_{t+1}} - q_t \right).
$$

Since $q_t = 1/(1 + R_t)^2$, we have:

$$
(4.2) \quad r_{L,t} = (1 + R_t)^2 \left( \frac{1}{1 + r_{t+1}} - \frac{1}{(1 + R_t)^2} \right) \approx 2R_t - r_{t+1}.
$$

The risk premium for domestic long-term bonds, $\rho_{L,t}$, is defined to be the difference between the expected rate of return on holding long-term bonds for one period and the rate of return for short-term bonds. Thus, we have:

$$
(4.3) \quad \rho_{L,t} = E_t(r_{L,t}) - r_t \approx 2[R_t - \frac{1}{2}\{r_t + E_t(r_{t+1})\}]
$$

where $E_t$ is the expectation operator conditional on the information set in period $t$, $\Omega_t$. We assume that $\Omega_t$ includes the current and past values of $r_t, R_t, r^*_t, R^*_t$, and $s_t$, where $r^*_t$ and $R^*_t$ are the foreign short and long-term interest rates, respectively, and $s_t$ is the natural log of the exchange rate expressed in terms of the domestic currency.

The rate of return on holding foreign bonds for one period in terms of the domestic currency, $r_{F,t}$, is:

$$
(4.4) \quad r_{F,t} = r^*_t + s_{t+1} - s_t.
$$

Let $\rho_{F,t}$, the risk premium for foreign bonds, denote the difference between the expected rate of return on holding foreign bonds for one period and the rate of return for short-term bonds. Then,

$$
(4.5) \quad \rho_{F,t} = E_t(r_{F,t}) - r_t = r^*_t + E_t(s_{t+1}) - s_t - r_t.
$$

$^6$Because the foreign interest rate is assumed to be constant, the foreign short- and long-term bonds are perfect substitutes.

$^7$The future exchange rate that appears in 4.5 reflects the risk of exchange rate investors face in holding the foreign bonds for one period.
The model assumes that, at time $t$, a representative investor with a constant absolute risk aversion (CARA) utility function maximizes his expected utility of wealth at the beginning of the time $t + 1$ ($= W_{t+1}$) subject to a budget constraint:

$$
\max E_t \left( -e^{-kW_{t+1}} \right)
$$

subject to

$$
W_t = B_{S,t}^d + B_{L,t}^d + B_{F,t}^d
$$

where $k$ is the coefficient of absolute risk aversion, and $d$ denotes demand, so that domestic currency amounts invested in domestic short, long-term, and foreign bonds are $B_{S,t}^d$, $B_{L,t}^d$, and $B_{F,t}^d$, respectively. $W_t$ is the initial wealth at time $t$, and the value of investor’s assets at the beginning of time $t + 1$, $W_{t+1}$, satisfies:

$$
W_{t+1} = B_{S,t}^d (1 + r_t) + B_{L,t}^d (1 + r_{L,t}) + B_{F,t}^d (1 + r_{F,t}).
$$

In the partial equilibrium model, the stochastic processes for the interest rates are exogenously given, and the utility function is parameterized. The equilibrium exchange rate satisfies the foreign bonds market clearing condition, $B_{F,t}^s = B_{F,t}^s$, where $B_{F,t}^s$ is the supply of foreign bonds to the domestic residents. It is assumed to be equal to the cumulative current account balance and to follow the dynamic equation

$$
B_{F,t}^s = B_{F,t-1}^s + C_t.
$$

$C_t$ is the current account balance in period $t$ that satisfies:

$$
C_t = -a + b s_t + u_t,
$$

$^8$Interest received by holders of foreign bonds is ignored.
where $b$ is a positive number, and $u_t$ is a trade shock which is assumed to be white noise with variance $\sigma_u^2$.

Suppose that $W_{t+1}$ is normally distributed conditional on $\Omega_t$ and that the measure of the absolute risk aversion, $k$, is a positive constant. Under these assumptions, a representative investor’s optimization problem is equivalent to maximizing:

\[
\max_{\{B_{dF,t}, B_{dL,t}\}} E_t(W_{t+1}) - \frac{k}{2} \text{var}_t(W_{t+1}),
\]

where

\[
E_t(W_{t+1}) = W_t(1 + r_t) + B_{dL,t}^d(\rho_{L,t}) + B_{dF,t}^d(\rho_{F,t})
\]

and:

\[
\text{var}_t(W_{t+1}) = (B_{dL,t}^d)^2 \text{var}_t(r_{t+1}) + (B_{dF,t}^d)^2 \text{var}_t(s_{t+1})
\]

\[
-2(B_{dL,t}^d)(B_{dF,t}^d) \text{cov}_t(r_{t+1}, s_{t+1}).
\]

First order conditions with respect to $B_{dF,t}^d$ and $B_{dL,t}^d$ are, respectively:

\[
\rho_{F,t} - k(B_{dF,t}^d)\text{var}_t(s_{t+1}) + k(B_{dL,t}^d)\text{cov}_t(r_{t+1}, s_{t+1}) = 0
\]

and

\[
\rho_{L,t} - k(B_{dL,t}^d)\text{var}_t(r_{t+1}) + k(B_{dF,t}^d)\text{cov}_t(r_{t+1}, s_{t+1}) = 0
\]

Solving for $B_{dF,t}^d$ and $B_{dL,t}^d$ yields the demand functions for foreign bonds and domestic long-term bonds, respectively. In particular, the demand for the foreign bond is

\[
B_{dF,t}^d[\rho_{F,t}, \rho_{L,t}] = \psi \cdot \rho_{F,t} - \psi \cdot \phi \cdot \rho_{L,t}
\]
while the demand for the domestic long-term bond is:

\[ B_{L,t}^d[\rho_{F,t}, \rho_{L,t}] = -\psi \cdot \phi \cdot \rho_{F,t} + \psi \cdot \frac{\sigma_s^2}{\sigma_r^2} \cdot \rho_{L,t}. \]  

(4.16)

Here, \( \psi \) and \( \phi \) are \( 1/k \sigma^2 (1 - \text{cor}^2) \) and \( -\text{cov}/\sigma_r^2 \), respectively. \( \sigma_s^2 \) is the conditional variance of the exchange rate and is expressed by \( E_t[(s_{t+1} - E_t(s_{t+1}))^2] \) while \( \sigma_r^2 \) is the conditional variance on the short-term interest rate and is expressed by \( E_t[r_{t+1} - E_t(r_{t+1})]^2 \). The conditional covariance between the exchange rate and the short-term interest rate, \( \text{cov} \), is \( E_t[(s_{t+1} - E_t(s_{t+1}))(r_{t+1} - E_t(r_{t+1}))] \) while the correlation, \( \text{cor} \), is \( \text{cov}/(\sqrt{\sigma_r^2} \sqrt{\sigma_s^2}) \).

The demand function for foreign bonds, 4.15, depends on \( \text{cov} \), the covariance conditional on \( \Omega_t \) between the exchange rate and the short-term interest rate, and on \( \sigma_s^2 \), the conditional variance of the exchange rate. At the same time, the stochastic processes of the exchange rate and \( \text{cov} \) also rely on the demand function for foreign bonds. Therefore, in order to solve for a rational expectation equilibrium, the values of \( \text{cov} \) and \( \sigma_s^2 \) must be consistent with the stochastic process of the exchange rate implied by the demand function for foreign bonds.

When the short-term interest rate rises, there are two opposite effects on the demand for foreign bonds given the second moments of the exchange rate and the short-term interest rate. These effects are the direct and indirect risk premium effects we defined in Section 3. The direct risk premium effect is shown in the first term of 4.15. This effect is the change in the demand for foreign bonds when the short-term interest rates rise, holding the risk premium for long-term bonds constant. This effect is equal to \( -\psi \) and is negative. The indirect risk premium effect is the second term of 4.15. This effect is the change in the demand for foreign bonds when the short-term interest rate rises, holding the risk premium for foreign bonds constant. This effect
is equal to $\psi\phi$. In the rational expectations equilibrium derived in the next section, $cov$ is negative, which implies that the indirect risk premium effect is positive.

An intuitive explanation of the indirect risk premium effect is as follows: If the short-term interest rate unexpectedly rises, the price of a long-term bond falls and long-term bond holders suffer an unexpected capital loss. When $cov$ is negative, the exchange rate tends to appreciate causing investors to suffer an additional unexpected loss if they hold foreign bonds. Therefore, as long as an increase in the short-term interest rate is associated with an appreciation of the domestic currency, risk averse agents will want to avoid holding both long-term bonds and foreign bonds. When this association is stronger, investors are more willing to substitute between domestic long-term bonds and foreign bonds. In particular, when an increase in short-term interest rates reduces the risk premium for long-term bonds, risk averse investors want to adjust their portfolios toward holding more foreign bonds and fewer long-term bonds. This indirect risk premium effect allows the demand for foreign bonds to increase when the short-term interest rate rises.

The existence of two opposite effects on the demand for foreign bonds implies that the impact of a rise in the short-term interest rate on the demand for foreign bonds depends on the relative strengths of these two effects. The indirect risk premium effect dominates the direct risk premium effect if and only if $\phi > 1$. Therefore, $\phi$ could be thought of as the relative magnitude of the indirect risk premium effect. In the next section, it will be shown that $\phi$ is greater than 1 under reasonable parameter configurations.

The intuition for indirect and direct risk premium effects can be generalized with Ogaki’s (1990) concepts of direct and indirect substitution effects to the cases of other
utility functions and more than three assets. Given the second moments, 4.15 and 4.16 give the demand functions for foreign bonds and long-term bonds, respectively, as functions of expected returns. Hence it is possible to define substitution and income effects for changes in expected returns as in Blanchard and Plantes (1977) and Royama and Hamada (1967). Because absolute risk aversion is assumed to be constant, income effects do not appear in the demand for risky assets, notably foreign bonds and long-term bonds. Therefore, the price effects which appear in 4.15 and 4.16 are also the substitution effects. We can decompose the substitution effect into direct and indirect substitution effects. Even though the indirect substitution effect is not equal to the indirect risk premium effect, the indirect risk premium effect dominates the direct risk premium effect if and only if the indirect substitution effect dominates the direct substitution effect. As we show in the next section, the indirect risk premium effect dominates with reasonable parameter values in our model. In such cases, domestic short-term bonds and foreign bonds are complements. For the general utility function with any number of assets, a substitute of a substitute is always an indirect complement. In our model, foreign bonds and domestic long-term bonds are substitutes, and domestic long-term bonds and domestic short-term bonds are substitutes. This means that domestic short-term bonds and foreign bonds are indirect complements.

See Ogaki (1999), an earlier version of the present paper, for details.
5 The Rational Expectation Equilibrium

We use the model presented in Section 4 to derive the rational expectation equilibrium. The stochastic processes of interest rates are assumed to be as follows:

\[ r_t = \mu + e_t + \varepsilon_t, \]
\[ R_t = \frac{1}{2} d + \mu + \frac{1}{2}(1 + c)e_t, \]
\[ r^*_t = \mu, \]
and:
\[ R^*_t = \frac{1}{2} d + \mu, \]

where \( e_t \) is a persistent interest rate shock and \( \varepsilon_t \) is a temporary interest rate shock. It is assumed that \( e_t \) follows an AR(1) process:

\[ e_t = c e_{t-1} + v_t, \quad |c| < 1 \]

and that it is independent of \( u_t \). It is also assumed that \( \varepsilon_t \), and \( v_t \) are white noise with variance \( \sigma^2_e \) and \( \sigma^2_v \), respectively, and that they are independent of both each other and \( u_t \).\(^{10}\) Finally, \( d \) and \( \mu \) are positive numbers.

The conditional expectation is assumed to coincide with the best linear prediction. Since 5.2 is a fundamental representation in the sense of linear prediction theory (see e.g., Rozanov (1967)), observing the current and past values of \( R_t \) is equivalent

\(^{10}\)Whether \( v_t \) and \( u_t \) are independent with each other depends on the monetary policy regime as the central bank may change the interest rate in response to a trade shock. An example of a monetary policy regime in which these two shocks are independent is a forward-looking type of Taylor rule responding to both expected inflation and domestic expected output gap.
to observing the current and past values of $e_t$ under an AR(1) process. It follows that:

$$E_t(r_{t+1}) = \mu + ce_t,$$

and from 4.3 and 5.6,

$$\rho_{L,t} = d - \varepsilon_t.$$

For our purposes, we assume that the risk premium for long-term bonds is non-zero. As shown in 5.7, the assumption employed here is that only $e_t$ is transmitted to the long-term interest rate,\(^{11}\) so the risk premium is equal to the mean of the long-term interest rate plus a temporary interest rate shock.

Define $\eta = \sigma^2_{\varepsilon}/\sigma^2_{e}$, which may be interpreted as the measure of substitution between short-term bonds and long-term bonds. If $\eta = 0$, then the risk premium for long-term bonds will be the mean of the long-term interest rate, implying that the short-term bond and the long-term bond will become more substitutable. The greater the magnitude of $\eta$, the smaller the degree of the substitution will be.

Let $L$ be the lag operator. Then the equilibrium condition in period $t$ is,

$$E_t[A_0(L)s_t] = a + D_0,$$

where:

$$A_0(L) = -\psi L^{-1} + (b + \psi),$$

\(^{11}\)Ellingsen and Söderström(2001) suggest that the co-movement of term structure of interest rates depends on market participants’ interpretation of the policy move. They show that if a change in short-term rate is regarded as being caused by an unexpected shift in policy preference, the federal funds rate and the long term interest rates will move in opposite directions. Further, Kobayashi(2004) emphasizes that the simultaneous occurrence of economic shocks which have different signs and durations can break down the co-movement of term structure of interest rates.
and:

\[ D_0 = -u_t - B_{F,t-1}^s - \psi \phi d - \psi e_t + \psi (\phi - 1)e_t. \]  

(5.10)

Taking conditional expectations with respect to \( \Omega_t \) on both sides yields the equilibrium condition for period \( t + 1 \):

\[ E_t[A(L)s_{t+1}] = a + D_1, \]

(5.11)

where:

\[ A(L) = -\psi L^{-1} + (b + 2\psi) - \psi L, \]

(5.12)

and:

\[ D_1 = \psi (1 - c)e_t - \psi (\phi - 1)e_t. \]

(5.13)

Analogously, taking expectations conditional on \( \Omega_t \) on both sides yields the equilibrium condition for any period \( t = \tau \), where \( \tau \geq 2 \):

\[ E_t[A(L)s_{t+\tau}] = a + D_2, \]

(5.14)

where:

\[ D_2 = \psi (1 - c)e_t c^{\tau-1}. \]

(5.15)

Solving 5.8, 5.11, and 5.14 as a difference equation system of \( E_t(s_{t+\tau}) \) with respect to \( \tau \) provides the unique saddle point solution for \( s_t \):

\[ s_t = \bar{s} - \left( \frac{1 - \lambda}{b} \right) u_t - \left( \frac{1 - \lambda}{b} \right) B_{F,t-1}^s - \left( \frac{\lambda}{1 - \lambda c} \right) e_t + \lambda (\phi - 1)e_t, \]

(5.16)

where \( \bar{s} = \frac{a}{b} - (\frac{1 - \lambda}{b})\phi \psi d \) is the long-run equilibrium exchange rate that clears the current account, and

\[ \lambda = 1 + \frac{b}{2\psi} - \frac{b}{2\psi} \sqrt{1 + \frac{4\psi}{b}}. \]

(5.17)
It can be shown that $0 < \lambda < 1$, $\partial \lambda / \partial \psi > 0$, $\lim_{\psi \to 0} \lambda = 0$, and $\lim_{\psi \to \infty} \lambda = 1$.

Equation 5.16 shows that the investor’s expected values of $\text{cov}$ and $\sigma^2_s$ affect the exchange rate dynamics through $\lambda$ and $\phi$. On the other hand, the exchange rate dynamics in 5.16 imply certain values of $\text{cov}$ and $\sigma^2_s$, which need to be consistent with the investor’s expected values in the rational expectation equilibrium. To analyze the equilibrium, we first solve for the rational expectation of $\text{cov}$. We then show the uniqueness and existence of the equilibrium by solving for the rational expectation of $\sigma^2_s$.

Before solving for the equilibrium, note the nature of Equation 5.16. It explains the discrepancy between actual and long-term equilibrium exchange rates through four factors: the trade shock at period $t$, the cumulative current account balance, and the persistent and temporary interest rate shocks. The trade shock $u_t$ tends to give rise to a current account surplus, and thus makes domestic currency to appreciate. As the cumulative current account balance becomes greater, the domestic currency appreciates more rapidly. For investors to hold foreign bonds, domestic currency must appreciate in order for investors to anticipate that it will depreciate in the future. Prolonged increases in the short-term interest rate also make the domestic currency appreciate. All of these effects are consistent with the expected directions. However, the temporary interest rate shock, $\varepsilon_t$, has a positive effect if the relative magnitude of the indirect risk premium effect, $\phi$, is greater than one.

Calculating $\text{cov} = E_t[\{s_{t+1} - E_t(s_{t+1})\}\{r_{t+1} - E_t(r_{t+1})\}]$ from 5.1 and 5.16 yields:

$$\text{cov} = -(\frac{\lambda}{1 - \lambda c})(1 - c^2)\sigma_e^2 + \lambda(\phi - 1)\sigma_e^2.$$  

(5.18)
Substituting the definition of $\phi$ into 5.18, and solving for $cov$ gives the rational expectation equilibrium values for $cov$ and $\phi$:

\[
(5.19) \quad cov = -\left[\lambda(1-c^2) + \lambda \eta(1-\lambda c)\right] \left[\frac{1 - c^2 + \eta}{1 - c^2 + \eta(1+\lambda)}\right] \sigma_e^2 < 0,
\]

and:

\[
(5.20) \quad \phi = \left[\frac{\lambda(1-c^2) + \lambda \eta(1-\lambda c)}{1-\lambda c}\right] \left[\frac{1}{1 - c^2 + \eta(1+\lambda)}\right] > 0.
\]

In the rational expectation equilibrium, the conditional covariance between the exchange rate and the short-term interest rate, $cov$, is negative, and the measure of the relative magnitude of the indirect risk premium effect, $\phi$, is positive. This implies that the indirect risk premium effect is positive as shown in the previous section. The main issue for the purpose of this paper is whether $\phi$ is greater or less than one. In order to determine this, we will investigate the sign of

\[
(5.21) \quad \phi - 1 = \frac{(1 - c^2)\{\lambda(1+c) - 1\} - \eta(1-\lambda c)}{(1-\lambda c)\{1 - c^2 + \eta(1+\lambda)\}}.
\]

In order to examine the sign of 5.21, we need to know how $\lambda$ depends on the underlying parameters of the model. For this purpose, the existence and the uniqueness of the rational expectation equilibrium will be shown by solving for the rational expectation of the conditional variance of the exchange rate, $\sigma_s^2 = E_t[\{s_{t+1} - E_t(s_{t+1})\}^2]$. By taking a period lead operator of 5.16, we obtain:

\[
(5.22) \quad \sigma_s^2 = \left(1 - \frac{\lambda}{b}\right)^2 \sigma_u^2 + \left[\frac{\lambda^2(1-c^2) + \lambda^2(\phi - 1)^2 \eta(1-\lambda c)^2}{(1-\lambda c)^2}\right] \sigma_e^2;
\]

and by 5.19 and the definition of $cor$,

\[
(5.23) \quad cor = -\sqrt{\frac{(1-c^2 + \eta)\sigma_e^2}{\sigma_s^2}} \cdot \left[\frac{\lambda(1-c^2) + \lambda \eta(1-\lambda c)}{(1-\lambda c)(1 - c^2 + \eta(1+\lambda))}\right].
\]
Using the definition of $\lambda$ in 5.17 allows us to obtain:

\begin{equation}
\psi = \frac{b\lambda}{(1 - \lambda)^2},
\end{equation}

and substituting in the definition of $\psi$ into 5.24 gives:

\begin{equation}
\frac{1}{k} = \sigma^2_s(1 - \text{cor}^2) \frac{b\lambda}{(1 - \lambda)^2}.
\end{equation}

The condition for the rational expectation equilibrium value for $\sigma^2_s$ is obtained by substituting 5.22 and 5.23 into 5.25; in particular,

\begin{equation}
\frac{1}{k} = g(\lambda),
\end{equation}

where $g(\lambda) = \frac{\sigma^2_s\lambda}{b} + \frac{b\sigma^2_s\lambda^2(1 - \text{cor}^2)}{(1 - \lambda)^2(1 - \text{cor}^2)} - \frac{b\sigma^2_s(1 - \text{cor}^2)\lambda^2(1 - \text{cor}^2)}{(1 - \lambda)^2(1 - \text{cor}^2) + \eta(1 - \lambda)^2}.$

Let $\lambda^*$ be the value of $\lambda$ that satisfies (5.26). Any such $\lambda^*$ corresponds to a rational expectation equilibrium. It can be checked that $\lim_{\lambda \to 0} g(\lambda) = 0$, and $\lim_{\lambda \to 1} g(\lambda) = \infty$. In particular, under the parameter configuration employed in the following Monte Carlo simulation, it can be shown that $g'(\lambda) > 0$. Hence, there exists a unique rational expectation equilibrium. Moreover, when $k$ is smaller, $\lambda^*$ is larger.

It can be shown that $\lim_{k \to 0} \lambda = 1$ and $\lim_{k \to \infty} \lambda = 0$. The value of $\psi$ can be obtained by substituting $\lambda^*$ for $\lambda$ in (5.24). $\psi$ is decreased by both a reduction in the variances $\sigma^2_u$ and $\sigma^2_e$ and by an increase in the measure of constant risk aversion, $k$, which in turn diminishes $g(\lambda)$.

Equation 5.21 shows that $\phi$ can be either greater or less than one, depending on the parameter values. One interesting case arises when the investor is close to being risk neutral. For a very small $k$, an approximate formula for (5.21) with $\lambda \cong 1$ is:

\begin{equation}
\phi - 1 = \frac{(1 + c)(1 - \eta)}{1 - c^2 + 2\eta}.
\end{equation}
We investigate the conditions required to exhibit the forward premium anomaly under low degrees of risk aversion. The forward premium regression for short-term interest rate differential is:

\[
\begin{align*}
  s_{t+1} - s_t &= \alpha + \beta (r_t - r^*_t) + \text{error term} \\
  (5.28)
\end{align*}
\]

Let \(\hat{\beta}\) be the estimate of \(\beta\). The probability limit of estimator is:

\[
\begin{align*}
  \text{plim} \hat{\beta} &= \frac{\text{cov}(r_t - r^*_t, s_{t+1} - s_t)}{\text{var}(r_t - r^*_t)} \\
  &= \frac{\text{cov}(r_t, s_{t+1} - s_t)}{\text{var}(r_t)} \\
  (5.29)
\end{align*}
\]

For this to be negative, we need \(\text{cov}(r_t, s_{t+1} - s_t) < 0\), which implies:

\[
\begin{align*}
  \text{cov}(r_t, s_{t+1}) &< \text{cov}(r_t, s_t) \\
  (5.30)
\end{align*}
\]

However, substituting 5.27 into 5.30 does not produce a positive value of \(\eta\) that satisfies 5.30. Thus, the population limit of \(\hat{\beta}\) is always positive in this model. The persistent shock creates a positive slope coefficient because both short- and long-term interest rates move together in response to this shock in the model. The persistent shock dominates the temporary shock in the limit; however, given that the temporary shock can cause the slope to be negative due to the dominant indirect risk premium effect, there is likely to be a small sample bias when the exchange rate is persistent.

We conduct a Monte Carlo simulation\(^{12}\) under these parameterizations to investigate the possibility of the small sample bias. Because the model is highly stylized so that we can analytically solve for the rational expectation of the covariance, we do not try to calibrate the data in this paper.\(^{13}\)

\(^{12}\)Gauss for Windows XP Version 7.0.4 was used for this paper to conduct the simulation.

\(^{13}\)Baillie and Kilic (2006) employ the logistic smooth transition dynamic regression (LSTR) model to investigate some nonlinear and asymmetric aspects of the relationship between the exchange rate
Suppose that the AR(1) coefficient of the persistent interest rate shock, $c$, in 5.5 is close to one (for example, $c = 0.9$). Then, $\phi$ in 5.27 becomes greater than one as long as $\eta < 1.71$. When the investor is close to being risk-neutral, the degree of substitution between short and long-term bonds must be high, and consequently, $\eta$ should be very small. Under these parameter configurations, our model presented in the previous section predicts that when the measure of the relative magnitude of the indirect risk premium effect, $\phi$, is greater than one, the demand for foreign bonds increases as the short-term interest rate rises, resulting in the depreciation of domestic currency to cause an expected future appreciation of domestic currency. A Monte Carlo simulation based on these parameter configurations consistently generates a negative slope coefficient to show the forward premium anomaly. As Table 5.1 shows, the stronger the indirect risk premium effect, the more statistically significant the negative slope coefficient.

On the contrary, a Monte Carlo simulation for the long-term interest differential still generates, as Table 5.2 shows, a positive slope coefficient under the same parameter configurations as the standard exchange rate model predicts.

6 Conclusion

We derive the demand function for foreign bonds endogenously by solving for the rational expectation equilibrium and investigate how a rise in the short-term interest rate affects the demand for foreign bonds. It generates two opposite effects on the demand for foreign bonds. The direct risk premium effect comes from the and the short-term interest rate differential. They show that the stylized facts of the forward premium anomaly can be obtained from calibrating a data generating process from the estimated LSTR model as long as transaction costs from closing arbitrage conditions in financial market are large relative to potential gains.
Table 5.1: A Monte Carlo Simulation of the Slope Coefficient for the Short-Term Regression

\[(s_{t+1} - s_t) = \alpha + \beta (r_t - r^*_t) + \text{error term} \]

\[H_0: \beta = 0\]

<table>
<thead>
<tr>
<th>(\phi)</th>
<th>1.6352</th>
<th>2.7848</th>
<th>3.5593</th>
<th>5.1282</th>
<th>9.0952</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\eta = 0.7))</td>
<td>((\eta = 0.3))</td>
<td>((\eta = 0.2))</td>
<td>((\eta = 0.1))</td>
<td>((\eta = 0.01))</td>
<td></td>
</tr>
</tbody>
</table>

| \(\text{plim} \hat{\beta}\) | 0.3266 | 0.3573 | 0.4067 | 0.5338 | 0.9099 |
|\(\text{mean of } \hat{\beta}\) | 0.3843 | -0.7719 | -1.5508 | -3.1287 | -7.1185 |
|\(\text{mean of } \hat{\beta}\) | 0.1925 | -0.3868 | -0.7771 | -1.5677 | -3.5668 |
|\text{negative frequency} \((1)\) | 38.6 | 69.9 | 84.9 | 96.8 | 99.9 |
|\text{5 \% significance level} \((2)\) | 1.7 | 8.3 | 18.6 | 49.8 | 92.1 |
|\text{(10 \% significance level) \((3)\)} | (3.4) | (14.7) | (27.4) | (60.7) | (96.5) |

Note: 1) sample covariance  
2) percentage of negative coefficients among total iteration (=1,000)  
3) percentage of total iterations (=1,000) that reject \(H_0\) at a five percent significance level. Numbers in parentheses are that of ten percent significance level.  
4) sample size is 102 and \(c = 0.9\)

The fact that risk averse agents with short investment time-horizons want to reduce the demand for foreign bonds to increase the amount invested in risk-free assets. On the other hand, investors have another incentive: the indirect risk premium effect, to increase the demand for foreign bonds to minimize potential capital losses resulting from holding both foreign bonds and long-term domestic bonds.

We show that, under reasonable parameter configurations, the indirect risk premium effect is quantitatively important. In fact, the indirect risk premium effect can
Table 5.2: A Monte Carlo Simulation of the Slope Coefficient for the Long-Term Regression

\[(s_{t+2} - s_t) = \alpha + \beta(R_t - R^*_t) + \text{error term}\]

\[H_0 : \beta = 0\]

<table>
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<td>((\eta = 0.01))</td>
<td></td>
</tr>
</tbody>
</table>

| \(\text{plim } \hat{\beta}\) | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| \(\text{mean of } \hat{\beta}\) | 3.5283 | 3.5327 | 3.5357 | 3.5418 | 3.5571 |
| \(\text{mean of } \hat{\beta}\) | 0.9785 | 0.9793 | 0.9798 | 0.9809 | 0.9837 |
| \(\text{positive frequency}^{2)}\) | 90.9 | 90.4 | 89.7 | 88.8 | 85.5 |
| \(\text{5 \% significance level}^{3)}\) | 29.8 | 28.9 | 27.8 | 25.2 | 19.3 |
| \(\text{(10 \% significance level)}^{3)}\) | (40.4) | (40.2) | (39.4) | (36.0) | (28.0) |

Note: 1) sample covariance
2) percentage of positive coefficients among total iteration (=1,000)
3) percentage of total iteration (=1,000) that reject \(H_0\) at a five percent significance level.
Numbers in parentheses are that of ten percent significance level.
4) sample size is 102 and \(c = 0.9\)

dominate the direct risk premium effect, causing demand for foreign bonds to increase.
Combining the dominance of the indirect risk premium effect with a small-sample bias solution causes our model to be consistent with the stylized facts regarding the forward premium anomaly. A Monte Carlo simulation for short-term regressions based on this parameter specification consistently shows that the dominant indirect risk premium effect is likely to cause downward small sample bias when the exchange rate is persistent. The stronger the indirect risk premium effect, the more statistically
significant the negative slope coefficient. In this case, the forward premium anomaly about the short-term interest rate can be explained; the domestic currency depreciates now, creating expected future appreciation of the currency. For the long-term interest rate differential, this model still shows the same prediction on the exchange rate as standard exchange rate models. Byeon and Ogaki (1999) find such results for many of the G7 countries with cointegrating regressions of real exchange rates onto both short- and long-term interest rate differentials. Ogaki and Santaella (2000) obtain similar results for Mexico.

If the indirect risk premium effect is quantitatively significant, then the effectiveness of central banks’ attempts to affect the exchange rate by controlling the short-term interest rate depends on whether the long-term interest rate responds to changes in the short-term interest rate. Anecdotal evidence suggests that further empirical investigation is warranted. For example, from the middle of March 1982 to the end of November 1982, the Bank of Japan adopted a policy to increase the domestic short-term interest rate in order to cause an appreciation of the yen (see e.g., Komiya and Suda [1983, pp. 347-354]). The short-term interest rate in Japan increased, but the yen tended to depreciate rather than appreciate against the U.S. dollar during this period. One remarkable fact was that the long-term interest rate did not increase when the Bank of Japan began to increase the short-term interest rate (Komiya and Suda [1983, p.349]).

The model in this paper suggests that a much more complicated relationship might exist between the term structure of interest rates and the exchange rate than is implied by exchange rate models with risk-neutral agents. In addition, the model can be applied to the relationship between the exchange rate and the term structure
of various short-term rates if the investment horizon is very short (e.g., one month or shorter). In this sense, the model could help explain Clarida and Taylor’s (1997) finding that the information given by the term structure of forward premiums, which is based on months of up to 12 months, is useful in predicting the future exchange rate.

In typical VAR with the recursive identification assumption, the delayed overshooting puzzle has been found as in Eichenbaum and Evans (1995). One aspect of the delayed overshooting puzzle is that the Uncovered Interest Parity is severely violated. Therefore, the indirect risk premium effect in principle can help resolving the puzzle. Incorporating the indirect risk premium effect into an open economy general equilibrium model to see whether or not the delayed overshooting arises in such a model is of interest.

In this paper, we develop a highly stylized partial equilibrium model to obtain the rational expectation of the covariance between the exchange rate and the short-term interest rate, which is a key parameter for the indirect risk premium effect. It is of interest to study if the qualitative implications of the model still hold in more realistic models. There has been relatively little empirical work on the interaction between exchange rate and the term structure of interest rates. Further empirical investigation is warranted on this complicated interaction.

\[14\] It should be noted that other identification assumptions are consistent with much less duration of the delay or even no delay as in Kim and Roubini (2000), Faust and Rogers (2003), Jang and Ogaki (2004), and Kim (2005).
REFERENCES


and The Term Structure of Interest Rates, OSU Working Paper 20, The Ohio State University.


