Does the Liquidity Effect Guarantee a Positive Term Premium?

Forthcoming in “Economic Modelling”

Kyuil Chung*

The views expressed herein are those of the author and do not necessarily reflect the official views of the Bank of Korea. When reporting or citing it, the author’s name should always be stated explicitly.

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Does the Liquidity Effect Guarantee a Positive Term Premium?

This paper examines the liquidity effect and the term structure in two versions of the limited participation model — an imperfect information model and an adjustment cost model. With a discrete state solution approach, I find a striking contrast: while the imperfect information model successfully generates the liquidity effect and the positive term premium seen in the data; the adjustment cost model replicates only the liquidity effect. This is because the adjustment cost that drives the liquidity effect in the adjustment cost model also creates an adjustment cost effect, which leads to a negative term premium.

**Keywords:** term premium, the liquidity effect, limited participation model  
**JEL Codes:** E43, E52, E41
1 Introduction

As discussed in Blinder (1998) and Walsh (2003), monetary policy has become the central tool of macroeconomic stabilization, and most central banks use interest rates as their policy instruments. This interest rate-oriented monetary policy requires that we put more emphasis on the liquidity effect and the term structure — the cores of the monetary transmission mechanism. This paper examines the relationship between the liquidity effect and the term structure in two versions of the limited participation model — an imperfect information model due to Christiano, Eichenbaum, and Evans (1997a) and an adjustment cost model by Dotsey and Ireland (1995). According to the previous studies, if a monetary model is successful in generating a liquidity effect, the model also captures the positive term premium seen in the data. Here, the positive term premium means not the response of term premium to external shocks, but the sign of term premium. In other words, the yield curve is, on average, upward-sloped. For example, Backus, Gregory, and Zin (1989) and Salyer (1990) show that the positive term premium cannot be replicated in cash-in-advance (CIA) models which fail to capture the liquidity effect. Instead, Coleman, Gilles, and Labadie (1992) find that a limited participation model (LPM) which succeeds in reproducing the liquidity effect also generates a positive term premium.

While the limited participation model (LPM) successfully captures the liquidity effect, there are two sub-models in that category. The original limited participation models studied by Lucas (1990), Christiano and Eichenbaum (1991), and Fuerst (1992) impose a restriction on the timing of households’ decisions on cash holdings used for consumption. Unlike the timing in the CIA model, in which all the decisions are made after the money shock, in the original limited participation models, households are assumed to make their cash holding decisions before observing current

---

1 Among the extensive empirical literature showing the liquidity effect, Leeper and Gordon (1992), Strongin (1995), Leeper, Sims, and Zha(1996), Christiano, Eichenbaum, and Evans (1999) are orthodox examples.

money growth realization. Since its development, this modeling strategy has been widely used by Christiano and Eichenbaum (1992), Dow (1995), Christiano, Eichenbaum, and Evans (1997a), and others.3

Dotsey and Ireland (1995), on the other hand, argue that it is more realistic to assume that households can, at some cost, adjust their cash balances after observing true money growth realization. They insist that their model nests the CIA model and the original limited participation model: when the parameter is zero, their model shows the CIA model-behaviors implying no liquidity effect; as the size of the adjustment cost parameter increases, it succeeds in generating a liquidity effect; when the parameter is infinite, it behaves exactly the same as the original limited participation model. Cooley and Quadrini (1999), Hendry and Zhang (2001), and Keen (2004) follow this line, assuming that households’ cash holding decisions occur after money growth realization.

Though both models are based on the limited participation model setting, imperfect information (one-period-ahead decision making) is the critical factor in the imperfect information model, whereas it is adjustment cost in the adjustment cost model. This paper is motivated by this point. While both models show the liquidity effect, do they still show a positive term premium? In other words:

Does the liquidity effect guarantee a positive term premium?

To analyze this, we introduce Evans and Marshall’s (1998)4 bond market into the original CEE (1997a) model which lacks the analysis of term structure. We also use a discrete-state solution approach. The main reason for adopting this approach is that it allows me to capture the exact behavior of time-varying term premia, which are approximated in the linearization method.

Our results show a striking contrast: while the imperfect information model successfully replicates the liquidity effect and the positive term premium seen in the data; the adjustment cost model generates the liquidity effect but fails to capture the

---


4 Evans and Marshall (1998) analyze the response of term premium to a contractionary money shock, but this paper investigates the sign of term premium.
positive term premium. The main reason for these results is this. The liquidity effect in the adjustment cost model is driven by the adjustment cost. In addition, the adjustment cost of a one-period bond is always bigger than that of a two-period bond,\footnote{While the two-period bond return is purely determined in the bond market, the one-period bond return is affected by bond market and labor market conditions. Therefore, it takes additional labor for a household to adjust a one-period bond. A more detailed explanation will be given in Section 3.} which is dubbed the adjustment cost effect. Given these two conditions, the increase in the adjustment cost creates two effects: strengthening of the liquidity effect on the one hand and an even greater strengthening of the adjustment cost effect on the other hand. This means that if the parameter is small, the liquidity effect dominates the adjustment cost effect; but if the parameter is large, the situation is reversed. To generate the liquidity effect, on the whole, a large adjustment cost parameter is needed. However, with this big parameter value, the adjustment cost effect dominates the liquidity effect, leading to a negative term premium. Despite the virtue of the adjustment cost model – generation of the liquidity effect without the strong assumption of imperfect information – it has a weakness in capturing the correct term premium sign. In other words, the presence of liquidity effect does not guarantee the positive term premium sign.

We will briefly introduce a term premium theory in Section 2 and set up the two models in Section 3. The simulated results will be described in Section 4. The paper rounds off with some conclusions in Section 5.

2 A Basic Framework of Term Premium Theory

2.1 A Characterization of Uncertainty

In order to expedite our discussion in later sections, first I need to introduce the term premium determination theory in a simple setting. Since the focus of this paper is the relationship between the liquidity effect and the term premium, I assume that the only exogenous uncertainty comes from the random growth rate of money, $x_t$.\footnote{While the two-period bond return is purely determined in the bond market, the one-period bond return is affected by bond market and labor market conditions. Therefore, it takes additional labor for a household to adjust a one-period bond. A more detailed explanation will be given in Section 3.}
following Evans and Marshall (1998). Despite the fact that many central banks are now adopting interest rates as their operational tools, money growth rate shocks are essential here to investigate the liquidity effect: the increase of money growth rate results in the fall of short term interest rate. It is also assumed that the money growth rate follows a three-state Markov process. The realizations of $x_t$ can take on low, middle, or high values, denoted $s = 1, 2, \text{ and } 3$, respectively.

$$
\begin{align*}
&x_3 = \mu_x + \delta \\
&x_2 = \mu_x \\
&x_1 = \mu_x - \delta
\end{align*}
$$

(2-1)

Here, $\mu_x$ is the average money growth rate, and $\delta$ is the standard deviation of the $\mu_x$. Hence, a transitional probability matrix, which shows the probability that the money growth rate moves from the state of $t-1(t)$ period to the state of $t(t+1)$ period, is a three by three matrix. For example, in the probability matrix (2-2), $\pi_{11}$ and $\pi_{22}$ show the conditional probability that money growth rate moves from a low state to a low state, a middle state to a middle state, respectively. Under the assumption of uniform ergodic distribution of money growth rates, the unconditional probabilities, $(p1, p2, p3)$, are equal to $\frac{1}{3}, \frac{1}{3}, \text{ and } \frac{1}{3}$, respectively. If I assume normal ergodic distribution, $(p1, p2, p3)$ depend on the serial correlation parameter of money growth rates, as we will see in section 4.7.

$$
\Pi = \begin{pmatrix}
\pi_{11} & \pi_{12} & \pi_{13} \\
\pi_{21} & \pi_{22} & \pi_{23} \\
\pi_{31} & \pi_{32} & \pi_{33}
\end{pmatrix}
$$

(2-2)

Following this characterization of uncertainty, the movement from state one to state

---


7 See 4.1 parameterization.
two or state three means an expansionary money shock and vice versa. By comparing the value of economic variables, we can check each variable’s response to a money shock.

2.2 The Liquidity Effect

Before exploring the term premium sign, let’s check the liquidity effect in a CIA model. The presence of the liquidity effect is essential to determine the term premium sign. In a traditional CIA model, the FOC’s for one- and two-period bonds can be expressed like below:

\[
\beta R_{i}^{1} E_{t} \left[ \frac{U_{C,t+1}'}{P_{t+1}} \right] = \frac{U_{C,t}'}{P_{t}} \tag{2-3}
\]

\[
\beta (R_{i}^{1})^{2} E_{t} \left[ \left( \frac{U_{C,t+1}'}{P_{t+1}} \right) \frac{1}{R_{i+1}'} \right] = \frac{U_{C,t}'}{P_{t}} \tag{2-4}
\]

Here, \( \beta \) means a representative agent’s subjective discount rate. \( R_{i}^{1} \) and \( R_{i}^{11} \) are the gross returns of one- and two-period bonds at period \( t \), respectively. \( U_{C,t}' \) implies the marginal utility of consumption, and \( P_{t} \) is the price level. Note that the two-period bonds are liquidated after one period as shown in \( (R_{i}^{1})^{2} E_{t} \left[ \frac{1}{R_{i+1}'} \right] \) term in equation (2-4).

From equation (2-3), I can derive an equation which determines the movement of the gross return of one-period bonds:

\[
R_{i}^{1} = \frac{1}{\beta} E_{t} \left[ U_{C,t}' \frac{P_{i+1}}{P_{t}} \right] \tag{2-5}
\]

As is clear in equation (2-5), the movement of one-period nominal bond return is determined by intertemporal marginal utilities of consumption and expected inflation effects. We call this relationship *Fisherian Fundamentals*.

If we recall the previous assumption that there is only a money shock—
consumption growth is zero, then equation (2-5) implies that one-period nominal bond return is entirely determined by the expected inflation effect:
\[ R^i_t = \frac{1}{\beta} E_t\left[\frac{P_{t+1}}{P_t}\right] = \frac{1}{\beta} E_t[1 + \pi_{t+1}] \].

Under the assumption of a positive serial correlation of money growth rates (\( \pi > 1/3 \)), a realization of high money growth in this period implies that a high money growth state in next period is more likely. This raises the expected inflation. These whole mechanisms hint that the nominal bond return in the high state of money growth is bigger than that in the low state, \( R^1_t < R^2_t < R^3_t \) when \( \pi > 1/3 \).

Thus, the CIA model does not show the liquidity effect. With the same logic, I can express the movement of two-period nominal bond return like below by using equation (2-3) and (2-4):

\[ (R''^i_t)^2 = \frac{1}{\beta} E_t[R^i_{t+1} \frac{P_{t+1}}{P_t}] \] (2-6)

As is clear from equation (2-5) and \( R^i_t = \frac{1}{\beta} E_t[1 + \pi_{t+1}] \), two-period nominal bond return and an expansionary money shock show a positive relationship due to the assumption of the positive serial correlation of money growth rates, \( \pi > 1/3 \). This implies there is no liquidity effect in the CIA model. As will be discussed later, if we introduce new frictions such as imperfect information or adjustment cost into the CIA model, we can get the liquidity effect.

2.3 Term Premia

---

8 As long as nominal bond returns are positive, the CIA constraint is binding, \( M_t = P_t C_t \). Therefore, under the assumption of constant consumption, money growth rate, \( \chi_t \), is equal to inflation rate, \( \pi_t \).

9 If we extend the CIA model to a production economy, introducing leisure into the utility function additionally, money has a real effect in the short-run. The inflation tax on consumption which stems from a CIA constraint reduces consumption and increases a demand for leisure. This leads to the fall of current consumption and labor supply. Therefore, a positive money shock results in a fall of output, which is also contrary to the liquidity effect. For more discussions, refer to Walsh (2003) Ch. 3.
A term premium (conditional on the state of time t) is defined as the difference between the expected nominal return of two-period bond liquidated after one period and the certain nominal return of one-period bond:

\[
TP_t = (R_{t+1}^H)^2 E_t \left[ \frac{1}{R_{t+1}} - R_t^1 \right] 
\]  

(2-7)

As I mentioned in section 1, the empirical studies show that the sign of term premium is positive.\(^{10}\)

To check the sign of term premia, equation (2-7) can be transformed into

\[
TP_t = (R_{t+1}^H)^2 \left[ E_t \left( \frac{1}{R_{t+1}} - \frac{R_t^1}{(R_t^1)^2} \right) \right] 
\] 

for a computational convenience. Now, I can transform \( \frac{R_t^1}{(R_t^H)^2} \) into \( \frac{E_t[U_{C,t+1}']}{P_{t+1}} \) by using equation (2-3) and (2-4).

Finally the term premium equation is simplified like below:\(^{11}\)

\[
TP_t = (R_{t+1}^H)^2 \left\{ \frac{\text{Cov}_t \left[ \frac{U_{C,t+1}'}{P_{t+1}}, \frac{1}{R_{t+1}} \right]}{E_t \left[ \frac{U_{C,t+1}'}{P_{t+1}} \right]} \right\} 
\]  

(2-8)

It is obvious that the conditional covariance in equation (2-8) determines the term premium signs. Let’s assume that a high money growth realization in t+1 period. This

---

\(^{10}\) Equation (2-7) means the term premium is defined as a holding premium. If we define the term premium as a rolling premium, \( TP_t = (R_{t+1}^H)^2 - E_t[R_{t+1}^H | R_t^1] \). In any case, data show that the term premium is positive.

\(^{11}\) The intermediate processes are following:

\[
E_t \left( \frac{1}{R_{t+1}} - \frac{R_t^1}{(R_t^H)^2} \right) = E_t \left( \frac{1}{R_{t+1}} \right) - \frac{E_t(U_{C,t+1}')}{P_{t+1}} \frac{1}{R_{t+1}} = \frac{E_t(U_{C,t+1}')}{P_{t+1}} \frac{1}{R_{t+1}} - \frac{E_t(U_{C,t+1}')}{P_{t+1}} \frac{1}{R_{t+1}} 
\]
implies that the first term of covariance, \( \frac{U_{t+1}}{P_{t+1}} \), is a low state. Since money growth
rates are positively correlated, the money growth rate in t+2 period is also high, which
leads to a high \( R_{t+1}' \) as confirmed in equation (2-5). This implies that here is no
liquidity effect. Consequently, the covariance is positive and the term premium is
negative. In summary, a CIA model that does not capture the liquidity effect cannot
generate the positive term premium. Therefore, it seems that the liquidity effect is
essential to get the right sign of term premium. We will explore if this is always true in
the next section.
3 Model

3.1 The Imperfect Information Model

I borrow Christiano, Eichenbaum, and Evans (1997a)’s model and add to it Evans and Marshall’s (1998) bond market. Like Evans and Marshall (1998), I assume one-period bonds purchased at period t-1 are paid off at the end of period t-1, not at the beginning of period t. I follow this timing convention because the bond payoff is known with certainty and this ensures that all cash is kept in households at the end of each period.

Regarding the two-period bonds, I assume that two-period bonds bought at period t-1 are liquidated after one period. I also assume that the bond market precedes the goods market. The timing of events for the whole economy and the sequential flow of funds for each economic agent are depicted in Figure 1 and Figure 2.

3.1.1 Households

Households enter each period with $M_t$ units of money, $K_t$ units of capital, and $B_{t-1}^{II}$ units of two-period bonds purchased at period t-1. To focus on the short-run bond return behavior, I assume that all households own the same amount of capital, one unit. Through all the periods, therefore, $K_t = 1$.12 Before observing the money growth shock, households decide what amount of cash, $Q_t$, they will carry for consumption purchases. They, then take the remaining cash balance, $M_t - Q_t$, and two-period bonds to the financial intermediaries. At this point, the money growth shocks are realized. Upon observing the real money growth, households adjust their portfolios in the bond market by liquidating the two-period bonds bought at period t-1, then buy new one- and two-period bonds by using the cash balance, $M_t - Q_t$, and the

---

12 Considering that we are focusing on the liquidity effect, short-run movements of nominal interest rates in response to a monetary shock, this assumption is not irrelevant.
Figure 1: Timing of Events
(Imperfect Information Model)

Figure 2: Flow of Funds
(Imperfect Information Model)

[Households]

\[
\begin{align*}
\text{Inflow} & : M_t, (K_t, B_{t-1}^H, N_t) \\
B_{t-1}^I & : \left(\frac{R_{t-1}^I}{R_t} \right) \\
W_t, N_t & : B_t^I, R_t^I, r_t, K_t, F_t \\
\text{Outflow} & : M_t - Q_t, (B_{t-1}^H)
\end{align*}
\]

[Financial Intermediaries]

\[
\begin{align*}
\text{Inflow} & : M_t - Q_t, X_t \\
R_t, W_t, N_t & : B_t^I, R_t^I, F_t \\
\text{Outflow} & : W_t, N_t \\
\end{align*}
\]

[Firms]

\[
\begin{align*}
\text{Inflow} & : (N_t, K_t), W_t, N_t \\
P_t, Y_t & : W_t, N_t \\
\text{Outflow} & : W_t, N_t, R_t, W_t, N_t, r_t, K_t
\end{align*}
\]
liquidated two-period bond return, \( B_{t-1}^R \left( \frac{R_{t-1}^R}{R_t^R} \right)^2 \). This portfolio constraint is:

\[
B_t^1 + B_t^R \leq (M_t - Q_t) + B_{t-1}^R \left( \frac{R_{t-1}^R}{R_t^R} \right)^2
\]  

(3-1)

After completing the bond market transactions, households choose their consumption, \( C_t \), and labor, \( N_t \), to maximize their expected lifetime utility:

\[
E_0 \sum_{t=0}^{\infty} \beta^t [U(C_t, N_t)]
\]

It is assumed that the utility function has the following form:

\[
U(C_t, N_t) = \frac{[C_t - \frac{\psi_0}{1 + \psi_0} N_t^{1+\psi}]^{1-\gamma}}{(1-\gamma)}
\]

Here, \( \psi_0 \) is a scaling parameter, \( \psi \) is the inverse of labor supply elasticity, and \( \gamma \) is a risk-aversion parameter. At this point, firms pay their wage bills, \( W_t N_t \), in advance of production, and this labor income increases households’ total amount of money for consumption purchases in the goods market. This flow of funds for households’ consumption is expressed as the following CIA constraint:

\[
P_t C_t \leq Q_t + W_t N_t
\]

(3-2)

At the end of each period, households earn the rental profits from the firms, \( r_i K_t \), the returns on one-period bonds, \( B_t^1 R_t^1 \), and the total profits of the financial intermediaries and the firms, denoted \( F_t \) and \( D_t \), respectively. The following budget constraint shows households’ total flow of funds:

\[
13 \text{ This utility function ensures a constant labor supply elasticity, } \frac{1}{\psi}, \text{ with respect to a real wage.}
\]

11
Households’ optimal choice of $Q_t$, $C_t$, $N_t$, $M_{t+1}$, $B_t^1$, and $B_t^II$ must satisfy the following first order necessary conditions (FOC’s). Here, $\lambda_t$, $\nu_t$, and $\xi_t$ express multipliers for the portfolio constraint, equation (3-1), the CIA constraint, equation (3-2), and the budget constraint, equation (3-3):

$$M_{t+1} \leq Q_t + W_t N_t - P_t C_t + B_t^I R_t^I + r_t K_t + F_t + D_t \quad (3-3)$$

$$Q_t : E_{t-1} \left[ \frac{\lambda_t}{P_t} \right] = E_{t-1} \left[ \frac{\nu_t + \xi_t}{P_t} \right] \quad (3-4)$$

$$C_t : U'_C, t = \nu_t + \xi_t \quad (3-5)$$

$$N_t : U'_N, t + (\nu_t + \xi_t) W_t = 0 \quad (3-6)$$

$$M_{t+1} : \beta E_t \left[ \frac{\lambda_{t+1}}{P_{t+1}} \right] = \frac{\xi_t}{P_t} \quad (3-7)$$

$$B_t^1 : \beta E_t \left[ \frac{\lambda_{t+1}}{P_{t+1}} \right] = \frac{\lambda_t}{P_t} \quad (3-8)^{14}$$

$$B_t^II : \beta E_t \left[ \frac{\lambda_{t+1} R_t^{II}}{P_{t+1}} \right] = \frac{\lambda_t}{P_t} \quad (3-9)$$

The above FOC’s can be further simplified as below by using equations (3-4) and (3-5):

$$N_t : U'_N, t + U'_C, t \frac{W_t}{P_t} = 0 \quad (3-10)$$

$$B_t^I : \beta E_{t-1} \left[ R_t^I \frac{U'_C, t+1}{P_{t+1}} \right] = E_{t-1} \left[ \frac{U'_C, t}{P_t} \right] \quad (3-11)$$

---

^{14} Equation (3-8) is originally $\frac{\xi_t}{P_t} = \frac{\lambda_t}{P_t}$. If we combine this original one with equation (3-7), we can get equation (3-8).
The most important characteristic of the imperfect information model appears in equation (3-11). In that equation, the expectation operator is associated with $t-1$. This can be compared with one-period bond return equation (2-3) in a typical CIA model:

$$\beta R_t^f E_t[\frac{U'_{C,t+1}}{P_{t+1}}] = \frac{U'_{C,t}}{P_t}$$  \hspace{1cm} (2-3)$$

While equation (2-3) always holds, equation (3-11) in the imperfect information model holds only on average. This suggests that $R_t^f$ in the CIA model and $R_t^i$ in the imperfect information model may show different behaviors in response to a money shock.

Note that the two-period bond pricing FOC, equation (3-12), is also slightly different from equation (2-4) in the CIA model:

$$\beta (R_t^i)^2 E_t[\frac{U'_{C,t+1}}{R_{t+1}^i}] = \frac{U'_{C,t}}{P_t}$$  \hspace{1cm} (2-4)$$

3.1.2 Firms

Every period, firms hire capital and labor in order to maximize their profit. The maximization problem is:

$$\max_{k_t, n_t} P_t Y_t - R_t W_t N_t - r_t K_t$$

s.t. $Y_t = K_t^\alpha N_t^{1-\alpha}$

According to my assumptions on the firms’ flow of funds, firms have to pay their wage bills in advance of production. Hence, they must borrow their wage
payments, $W_t N_t$, from financial intermediaries. At the end of each period, firms must repay the wages borrowed multiplied by the gross nominal interest rate, denoted $R_t W_t N_t$, to the financial intermediaries and pay the rent for capital, $r_t K_t$, to the households.

Firms’ optimal choices of $N_t$ and $K_t$ must satisfy the following FOC’s:

$$R_t \frac{W_t}{P_t} = (1 - \alpha) \frac{(K_t)}{N_t}$$  \hspace{1cm} (3-13)

$$\frac{r_t}{P_t} = \alpha (\frac{K_t}{N_t})^{\alpha - 1}$$  \hspace{1cm} (3-14)

### 3.1.3 Financial Intermediaries

In every period, perfectly competitive financial intermediaries trade bonds with households at price unity. In addition to this bond trading, the financial intermediaries inelastically supply their funds, $M_t - Q_t + X_t$, to the firms in the loan market. Here, $M_t - Q_t$ is the net amount of money transferred from the households to the financial intermediaries and $X_t$ is the new money injection from the central bank. At the end of each period, the financial intermediaries obtain revenues, $R_t W_t N_t$, and pay households one-period bond returns, $B_t^i R_t^i$. A profit for the financial intermediaries, $F_t$ and the loan market clearing condition are given below:

$$F_t = M_t - Q_t + X_t - W_t N_t + R_t W_t N_t - B_t^i R_t^i$$  \hspace{1cm} (3-15)

$$W_t N_t = M_t - Q_t + X_t$$  \hspace{1cm} (3-16)

From equation (3-15), we know that the monetary injection, $X_t$, is distributed solely to the financial intermediaries; households are excluded from this money injection. The left-hand side of equation (3-16) represents the demand for funds, and
its right-hand side the supply of funds. Perfect competition among financial intermediaries ensures that the one-period nominal lending rate earned by the intermediaries, \( R_t \), is equal to the gross nominal return on one-period bonds paid by the intermediaries to the households, \( R^1_t \). This is:

\[ R_t = R^1_t \]  \( (3-17) \)

### 3.1.4 Central Bank

The central bank acts only as a money supplier to the financial intermediaries in this model. As mentioned earlier, to generate the liquidity effect, it is needed to assume that a central bank controls money supply as its policy instrument.

The evolution of money growth is defined as:

\[ M_{t+1} = (1 + x_t)M_t + X_t \]  \( (3-18) \)

The money growth rate, \( x_t \), follows a three-state Markov process explained in section 2.

### 3.1.5 Market Clearing Conditions

We can derive the market clearing conditions for the bond market, the goods market, the labor market, and the loan market. From the portfolio constraint, we can derive the net amount of one-period bonds bought at period \( t \). In equilibrium, generally, the net amount of one- or two-period bonds is equal to zero. In this limited participation model, however, due to the households’ sources of funds for buying new bonds \( (M_t - Q_1) \) and \( B^1_{t+1} \left( \frac{R^1_{t+1}}{R^1_t} \right) \), and to the timing of the pay-off for one-period bonds (the end of this period), the equilibrium amount of one-period bonds is \( M_t - Q_1 \), not

\(^{15}\) If we combine equations (3-15) and (3-16), we can derive the financial intermediaries’ profit, \( F_t = R_t W_t N_t - B^1_t R^1_t \),.
On the other hand, the equilibrium amount of two-period bonds is still zero. In this sense, two-period bonds are redundant assets. They have no effect on the equilibrium prices or quantities.\footnote{For more details on this modeling strategy, refer to Evans and Marshall (1998).}

The budget constraint implies the following goods market clearing condition:

\[ P_t C_t = P_t Y_t \]

The total amount of funds in the goods market is derived from the CIA constraint as follows:

\[ P_t C_t = M_t + X_t \] \hspace{1cm} (3-20)

From equation (3-10), (3-13), and (3-17), the labor market clearing condition is derived:

\[ R_t^1 = \frac{(1 - \alpha)}{\psi_0} N_t^{-\alpha - \psi} \] \hspace{1cm} (3-21)

For convenience, the loan market clearing condition of equation (3-16) is replicated here:

\[ W_t N_t = (M_t - Q_t) + X_t \] \hspace{1cm} (3-16)

If we combine equations (3-16) and (3-20), we can derive the following expression:

\[ \frac{W_t N_t}{P_t C_t} = \frac{(M_t - Q_t) + X_t}{M_t + X_t} \] \hspace{1cm} (3-22)

Equation (3-22) indicates the ratio of funds in the loan market to the funds in the goods market. In the imperfect information model, this equation is the most important one for deriving the liquidity effect. The wedge between the value of money in the loan market and the goods market is the critical source generating the liquidity effect.
This will be discussed in detail in Section 4. Here, I want to briefly show how it is related to the liquidity effect. The left-hand side of equation (3-22) can be rewritten as follows:\(^{17}\)

\[
\frac{(1-\alpha)}{R_t} = \frac{(M_t - Q_t) + X_t}{M_t + X_t}
\]  

(3-23)

Here, \(Q_t\) is the households’ money set aside for consumption before the money shock. Hence, \(M_t - Q_t\) is fixed before the new money injection, \(X_t\), is realized. Clearly, nominal interest rates and the new money injection show a negative relationship.\(^{18}\)

### 3.2 The Adjustment Cost Model

In the adjustment cost model, households make their cash holding decisions after observing the realized money growth state. However, there is a cost to them in adjusting their cash holding decisions. This adjustment cost is expressed as a certain amount of time, which is proportionate to the scaled cash balance.\(^{19}\) The critical

\(^{17}\) From equation (3-13), \(\frac{W_t}{P_t} = \frac{1-\alpha}{R_t} (N_t^{\omega})\). If we combine \(Y_t = K_t^{\alpha}N_t^{1-\alpha}\), \(K_t = 1\), and \(C_t = Y_t\), we can get \(C_t = N_t^{1-\alpha}\). Now, substituting \(\frac{W_t}{P_t} = \frac{(1-\alpha)}{R_t} (N_t^{\omega})\) and \(C_t = N_t^{1-\alpha}\) into the left-hand side of equation (3-22) yields equation (3-23).

\(^{18}\) \(M_t - Q_t\) in the numerator on the right-hand side of equation (3-23) is less than \(M_t\) in the denominator. Under this condition, the increase of \(X_t\) results in the rise of the whole term on the right-hand side of equation (3-23).

\(^{19}\) With no adjustment cost, households easily compensate for the money shock, \(X_t\), by adjusting cash holdings, \(Q_t\). Hence, there are no changes in the total money supply to the loan market by financial intermediaries, \(M_t - Q_t + X_t\). With the adjustment cost, households adjust their cash holdings by a small degree. If we instead introduce a fixed participation cost following Alvarez et al. (2002), households are divided into active ones with high or low cash holdings and inactive ones with intermediate cash holdings. Only the active ones respond to the money shock, while paying the fixed cost,
difference between the imperfect information model and the adjustment cost model lies in this timing of households’ cash holding decisions. This new timing of events and household flows of funds are depicted in Figure 3 and Figure 4. Note that, in Figure 4, \( M_t - Q_t \) occurs after the money growth realization.

3.2.1 Households

The households choose \( C_t \) and \( N_t \) in every period to maximize their expected lifetime utility. However, making the cash holding decision after the money growth realization costs the households some time for portfolio adjustment. This time cost of portfolio adjustment has a negative effect on households’ utility. Hence, the

by adjusting their cash balances, which leads to the liquidity effect. Over time, the active ones and the inactive ones switch their positions according to their idiosyncratic endowment shocks.
households’ new utility function is as follows:

\[
U(C_t, N_t, \frac{Q_t}{M_t}) = \left[ C_t - \frac{\psi}{1+\psi} \left\{ N_t + \frac{\phi}{2} \left( \frac{Q_t}{M_t} - \overline{q} \right)^2 \right\}^{1+\psi} \right]^{1-\gamma} / (1-\gamma)
\]

Here, \( \phi \) is an adjustment cost parameter and \( \overline{q} \) is a steady-state value of the cash balance. This implies that \( \frac{Q_t}{M_t} \) grows at \( \overline{q} \) rate in the steady state. Hence, in the steady state or under the condition that \( \phi \) is zero, the adjustment cost term vanishes.

Households’ portfolio constraint, CIA constraint, and budget constraint are the same as in the imperfect information model.

The households’ decisions must satisfy the FOC’s below:

\[
Q_t : \frac{\partial}{\partial t} = U_{Q,t} + \frac{U_{C,t}}{P_t} = 0
\]

\[
N_t : U_{N,t} + \frac{U_{C,t}}{P_t} = 0
\]

\[
M_{t+1} : \beta E_t [U_{M,t+1}] + \frac{\lambda_{t+1}}{P_t} = \frac{\xi_t}{P_t}
\]

---

20 I employ the quadratic adjustment cost term used by Dotsey and Ireland (1995).

21 $U'_{Q,t} = U_{C,t}(-\psi)\left\{ N_t + \frac{\phi}{2} \left( \frac{Q_t}{M_t} - \overline{q} \right)^2 \right\}^{\psi} \left( \phi \left( \frac{Q_t}{M_t} - \overline{q} \right) \frac{1}{M_t} \right)^{1-\psi}$

$U'_{M,t} = U_{C,t}(\psi)\left\{ N_t + \frac{\phi}{2} \left( \frac{Q_t}{M_t} - \overline{q} \right)^2 \right\}^{\psi} \left( \phi \left( \frac{Q_t}{M_t} - \overline{q} \right) \frac{Q_t}{M_t} \right)^{1-\psi}$
\[ B_{1}^{I} : \beta R_{t}^{I} \mathbb{E}_{t}[U_{M,t+1}^{I} + \frac{\lambda_{t+1}^{I}}{P_{t+1}}] = \frac{\lambda_{t}^{I}}{P_{t}} \quad (3-27) \]

\[ B_{1}^{II} : \beta \mathbb{E}_{t}[\frac{\lambda_{t+1}^{II} R_{t}^{II}}{P_{t+1} R_{t+1}^{II}}] = \frac{\lambda_{t}^{II}}{P_{t}} \quad (3-28) \]

The bond-pricing FOC’s above can be further simplified as below, by using equations (3-24) and (3-26):

\[ B_{1}^{I} : \beta R_{t}^{I} \mathbb{E}_{t}[\frac{U_{C,t+1}^{I} + U_{Q,t+1}^{I} + U_{M,t+1}^{I}}{P_{t+1}}] = \frac{U_{C,t}^{I} + U_{Q,t}^{I}}{P_{t}} \quad (3-29) \]

\[ B_{1}^{II} : \beta (R_{t}^{II})^{2} \mathbb{E}_{t}[\frac{U_{C,t+1}^{II} + U_{Q,t+1}^{II}}{P_{t+1}}] = \frac{U_{C,t}^{II} + U_{Q,t}^{II}}{P_{t}} \quad (3-30) \]

Again if we assume that \( \phi \) is zero, the \( U_{Q,t}^{I} \) and the \( U_{M,t}^{I} \) terms disappear, and equations (3-29) and (3-30) are exactly the same as equations in CIA model, (2-3) and (2-4).

The critical difference between the one-period bond return in the imperfect information model, equation (3-11), and that in the adjustment cost model, equation (3-29), is that while in the former case \( R_{t}^{I} \) is random, in the latter case \( R_{t}^{I} \) is not random. This means that while the former \( R_{t}^{I} \) is affected by the imperfect information; the latter \( R_{t}^{I} \) is distorted by the adjustment cost term.

\[ \frac{1}{P_{t}} \frac{\xi_{t}}{P_{t}} = \frac{\lambda_{t}}{P_{t}} \quad (3-27) \]

\[ 22 \text{This equation is originally } R_{t}^{I} \xi_{t} = \frac{\lambda_{t}}{P_{t}}. \text{ Like in the case of the imperfect information model, the discounted expected value of money is determined by the budget constraint multiplier. If we combine this original equation and equation (3-26), we can get equation (3-27). The reason why } U_{M,t+1}^{I} \text{ appears in equation (3-27) comes from the presence of the adjustment cost term, especially } \frac{Q_{t}}{M_{t}}, \text{ in the utility function. While } Q_{t} \text{ affects } \frac{\lambda_{t}}{P_{t}}, \text{ and } M_{t} \text{ affects } \frac{\xi_{t}}{P_{t}} \text{ as a } U_{Q,t}^{I}, \text{ and } \frac{\xi_{t}}{P_{t}} \text{ as a } U_{M,t+1}^{I}. \]

20
4 Results

4.1 Parameterization

To compute the equilibrium behaviors, parameter values must be calibrated. Since our interest is mainly in the bond return behaviors in the two models, I adopt parameter values from CEE (1997a).

The economic agents’ discount factor, $\beta$, is set to 0.9926. The elasticity of labor supply with respect to the real wage, $\frac{1}{\psi}$, is equal to 1. The parameter $\psi$, is calculated so that the steady state value of labor is 1. I use 0.36 as the capital share value of $\alpha$. Monetary policy behavior is characterized by $\delta$, $\mu$, and $\pi$. I assume that the money growth rate follows the three-state Markov process $(\mu_x - \delta, \mu_x, \mu_x + \delta)$. In an i.i.d. case, I set $\mu_x = 0.02$, and $\delta = 0.017$. When there is a positive correlation among money growth rates, I set $\mu_x = 0.02$, and $\delta = 0.02$. This means that the autocorrelation coefficient of the money growth rates is 0.5. The transitional probability $\mu_{jk} = \text{prob}\{x_{it} = x(k) | x_t = x(j)\}$, for $j, k = 1, 2, 3$, and $x (j)$ corresponds to $\mu_x - \delta$, $\mu_x$, and $\mu_x + \delta$, respectively, for $j=1, 2, 3$. In the i.i.d. case, I set $\mu_{jk} = 0.33$. 23

4.2 Equilibrium

Before defining equilibrium, we need to scale all nominal variables. In both models, $P_t, W_t, r_t, Q_t, X_t, B^I_t$, and $B^II_t$ are scaled by the beginning of the period money stock,

23 Following the CEE (1997a), I use $\mu_{jk} = \begin{pmatrix} 0.58 & 0.34 & 0.08 \\ 0.08 & 0.84 & 0.08 \\ 0.08 & 0.34 & 0.58 \end{pmatrix}$ in the positive correlation case. This transition probability matrix implies that the ergodic distribution follows a normal distribution. For more detailed discussion of the Markov process, refer to Christiano (1990).
M_t. The real variables \((C_t, N_t, K_t, Y_t)\), the nominal gross returns on one- and two-period bonds \((R^1_t, R^2_t)\), and the term premium \((TP_t)\) are not scaled.

### 4.2.1 The Imperfect Information Model

In this discrete-state solution approach, I need to clearly define each unknown’s state dependency. Currently, there are five unknowns, \(q_t, N_t, R^1_t, R^2_t, TP_t\). Here, \(q_t\) denotes the scaled amount of households’ cash holding, \(\frac{Q_t}{M_t}\). The decision to choose \(q_t\) is associated with period \(t-1\) information. Therefore, \(q_t\) takes on three states, \(q_j\), where \(j = 1, 2, 3\). All of the other variables are associated with both the results of the cash holding decisions occurring before the money growth realization \((j = 1, 2, 3)\) and with the conditions of the bond or goods markets which open after the money growth shock \((k = 1, 2, 3)\). Hence, each \(N_t, R^1_t, R^2_t\), and each \(TP_t\) takes on nine values, such as \(N_{j,k}, R^1_{j,k}, R^2_{j,k}\), and \(TP_{j,k}\), where \(j = 1, 2, 3\), and \(k = 1, 2, 3\). To get these 39 solutions, I construct 39 equations:

\[
\beta \mathbb{E}_j[R^1_{j,k} E_k[U'_C R^j_{k,l}]] = E_j[U'_C R^j_{k,l}]
\]

\[
\beta \frac{(R^2_{j,k})^2}{R^1_{j,k}} E_k[E_i[U'^C R^j_{i,m}]] = E_k[U'^C R^j_{i,m}]
\]

\[
R^1_{j,k} = \frac{(1 - \alpha)}{\psi_0} N^{-\sigma + \psi}_{j,k}
\]

\[
N^{\sigma + \psi}_{j,k} = \frac{1}{\psi_0} \left( 1 - q_j + x_k \right)
\]

\[
TP_{j,k} = (R^2_{j,k})^2 E_k[\frac{1}{R^1_{k,l}}] - R^1_{j,k}
\]

\[24\] Here, \(j\) denotes period \(t-1\) information, \(k\) period \(t\) information, \(l\) period \(t+1\) information, and \(m\) period \(t+2\) information.
Equation (4-3) is the labor market clearing condition. As is clear from this equation, the one-period bond return has an effect on real economic activity through the relationship $M_t - Q_t = B^1_t$ and $R_t = R^1_t$. The two-period bond return, however, appears only in the bond pricing formula equation (4-2), implying that $R^2_t$ has no real effect. Equation (4-4) is derived from the ratio of funds equation (3-22).\(^{25}\) Equation (4-5) is the definition of the term premium.

### 4.2.2 The Adjustment Cost Model

Unlike the case with the imperfect information model, all decisions in the adjustment cost model occur after the money growth realization. Hence, five unknowns take on only three values each, $k=1, 2, 3$. Therefore, fifteen non-linear equations are needed to get their solutions:

\[
\beta R^1_k E_k \left[ \frac{U'_{C,t} + U'_{Q,t} + U'_{M,t}}{P_t} \right] = \frac{U'_{C,k}}{P_k} + U'_{Q,k} \tag{4-6}
\]

\[
\beta (R^1_k)^2 E_k \left[ \frac{U'_{C,t} + U'_{Q,t}}{P_t} \frac{1}{R_t} \right] = \frac{U'_{C,k}}{P_k} + U'_{Q,k} \tag{4-7}
\]

\[
R_k^1 = \left(1 - \alpha \right) \frac{1}{\psi_0} N_k^{-\alpha} [N_k + \frac{\phi}{2} H_k^2]^{-\psi} \tag{4-8}
\]

\[
N_k^\psi [N_k + \frac{\phi}{2} H_k^2]^\psi = \frac{1}{\psi_0} \left( \frac{1 - q_k + x_k}{1 + x_k} \right) \tag{4-9}
\]

\[
TP_k = (R^1_k)^2 E_k \left[ \frac{1}{R_t} \right] - R^1_k \tag{4-10}
\]

\(^{25}\)To simplify the left-hand side of equation (3-22), we have to use equation (3-10), the labor supply curve ($\frac{W}{P_t} = \psi_0 N_k^\psi$). If we substitute the labor supply curve and the goods market clearing condition, $C_t = N_t^{\alpha}$, into the left-hand side of equation (3-22), and scale the right hand side of the equation, we get equation (4-4).
Here, $H_k = \left( Q_k - \bar{q} \right)$, the difference between the scaled cash balance and the steady-state value of it, and $\frac{\phi}{2} (H_k)^2$ is the portfolio adjustment cost term. Note that the adjustment cost term in the utility function has an effect on the total labor supply equation. While the imperfect information model’s labor supply curve is $\frac{W_t}{P_t} = \psi_t N_t^\psi$, it is expressed as $\frac{W_t}{P_t} = \varphi_t (N_t + \frac{\phi}{2} H_k^2)^\psi$ in the adjustment cost model. Since the portfolio adjustment process takes households’ time, the total amount of labor is modified to include this adjustment cost. This modified labor supply equation changes the labor market clearing condition, equation (4-8), and the ratio of funds in the loan market to the funds in the goods market, equation (4-9). Equations (4-1) to (4-5) and (4-6) to (4-10) are exactly symmetric except for the state dependent subscripts and the adjustment cost term in the total labor supply. Note that in the steady state, the adjustment cost is zero and each time subscript vanishes. This implies that equations (4-1) to (4-5) and equations (4-6) to (4-10) share the same equilibrium points.

4.3 The Liquidity Effect and Term Premia

The response of each variable to a contractionary money shock is calculated via the following elasticity concepts as in CEE (1997a):

$$dv = \log \left( \frac{V_{t+1}}{V_t} \right) / \log \left[ \frac{(1 + \mu_s)}{(1 + \mu_s - \delta)} \right]^{26}$$

$$dw = w_{t+1} - w_t / \log \left[ \frac{(1 + \mu_s)}{(1 + \mu_s - \delta)} \right]$$

26 Theoretically, $dv = \log \left( \frac{V_{t+1}}{V_t} \right) / \log \left[ \frac{(1 + \mu_s - \delta)}{(1 + \mu_s)} \right]$, when we analyze the responses of $v$ to a contractionary money shock. However, to get a negative sign in this case, I change the denominator to $\frac{(1 + \mu_s)}{(1 + \mu_s - \delta)}$. For the case of an expansionary money shock, we have to instead use $\frac{(1 + \mu_s + \delta)}{(1 + \mu_s)}$. 
Here \( v = (N_t) \), and \( w = (R^1_t, R^II_t, \text{INF}_t) \). Hence, \( dv \) represents a percentage change of each variable with respect to a one percent change of unanticipated money shock, and \( dw \) denotes the simple change of each variable which is scaled by \( \log[\frac{(1 + \mu_s)}{(1 + \mu_s - \delta)}] \).

Accordingly, if we look at the response of \( I^R_t \) and \( II^R_t \), we can check the presence of liquidity effect. To examine the sign of term premium, we have to see the value of term premium in each state (conditional) and calculate the average (or unconditional) value of term premium.

### 4.3.1 The Imperfect Information Model

The responses of economic variables to the money contraction in this model are summarized in Table 1. The first part of the table shows the case in which the labor-supply elasticity \( \frac{1}{\psi} \) is unity under different risk aversion parameters \( \gamma \). An i.i.d. case and a positive correlation case show the same pattern. To the contractionary money shock, the one- and two-period bond returns \( (R^1_t, R^II_t) \) rise — demonstrating the presence of the liquidity effect. Obviously, labor \( (N_t) \) falls and the inflation rate \( \text{INF}_t \) decreases. Notably, the response of \( I^R_t \) is greater than that of \( II^R_t \). Also, the responses of \( R^1_t, R^II_t \), and \( N_t \) in the i.i.d. case are greater than those in the correlation case, the reason being that the expected inflation effect is bigger in the correlation case. Unlike in the CIA model, the responses of \( R^1_t, R^II_t \), and \( N_t \) in this limited participation model are determined by the interaction of the liquidity effect, stemming from the wedge of money values between the money market and the goods market, and the expected inflation effect. In the i.i.d. case, the expected inflation effect is smaller than that in the correlation case. This causes the bigger responses of \( R^1_t, R^II_t \), and \( N_t \) in the i.i.d. case. Consequently, the response of \( \text{INF}_t \) is larger in the correlation case. The
risk aversion parameter \( (\gamma) \) has little effect on the different economic variables’ responses in either case.

The second part of the table exhibits the simulation results under different labor supply elasticities, \( \frac{1}{\psi} \). When the labor supply elasticity is very big \( (\psi = 0.1) \), this implies that households value leisure and real wages do not move much. Hence, \( N_t \) and \( \text{INF}_t \) respond more aggressively. When the labor supply elasticity is very small \( (\psi = 8) \), this implies that households do not value leisure and only real wages move. Accordingly, \( N_t \) and \( \text{INF}_t \) respond less aggressively. However, there is no qualitative difference between the benchmark case \( (\psi = 1, \gamma = 1) \) and the various simulation cases.

Table 1: Responses to a Money Contraction in the Imperfect Information Model

<table>
<thead>
<tr>
<th>Vars.</th>
<th>i.i.d.</th>
<th>Corr. (Coef. = 0.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \gamma = 1 )</td>
<td>( \gamma = 5 )</td>
</tr>
<tr>
<td>( R_1^t )</td>
<td>0.63</td>
<td>0.63</td>
</tr>
<tr>
<td>( R_2^t )</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>( N_t )</td>
<td>-0.45</td>
<td>-0.45</td>
</tr>
<tr>
<td>( \text{INF}_t )</td>
<td>-0.29</td>
<td>-0.29</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Vars.</th>
<th>i.i.d.</th>
<th>Corr. (Coef. = 0.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \gamma = 1 )</td>
<td>( \psi = 0.1 )</td>
</tr>
<tr>
<td>( R_1^t )</td>
<td>0.63</td>
<td>0.63</td>
</tr>
<tr>
<td>( R_2^t )</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>( N_t )</td>
<td>-1.33</td>
<td>-0.07</td>
</tr>
<tr>
<td>( \text{INF}_t )</td>
<td>-0.86</td>
<td>-0.05</td>
</tr>
</tbody>
</table>

Note: \( N_t \) is a percentage change, the other variables absolute changes.

---

27 By using equation (3-20) and \( Y_t = C_t \), we can derive \( P_t = \frac{M_t + X_t}{N_t^{1-\alpha}} \).
Term premium signs are shown in Table 2. The first part of the table is the benchmark case, and it shows all term premia to be positive. Due to the imperfect information setting, the term premium takes on nine values. The second and third parts of the table are various simulations under different labor supply elasticities. As in Table 1, there is no significant difference between the benchmark case and the various simulations. Obviously, the average term premium is positive in either the i.i.d. or the correlation case as we can see in Table 1.

To see the liquidity effect and the term premium signs from another angle, we can explore the FOC’s, equations (4-1) to (4-5), thoroughly. In equation (4-4), \( q_j \) is predetermined in each period due to the households’ making of portfolio decisions before the money shock. Under this condition, a new money shock, \( x_k \), causes an increase of the right-hand side of equation (4-4). Accordingly, the amount of labor, the left-hand side of (4-4), also goes up. This increased labor supply results in a decrease of the one-period bond return, \( R_{1,j,k} \), via equation (4-3). Hence, the liquidity effect is accomplished through equations (4-3) and (4-4).

Based on the liquidity effect, the one-period bond return and the two-period bond return are determined along with equations (4-1) to (4-3). Finally, the term premium definition, equation (4-5), produces the positive term premium result. One notable feature of this model is that the one-period bond return is affected by the bond market condition, equation (4-1), and the labor market condition, equation (4-3); the two-period bond return, however, is only determined by the bond market condition, equation (4-2).

At this point, we need to look at the labor market condition further. In the imperfect information setting, the key interest rate that is affected by the availability of money market funds is a lending rate. This lending rate is tied to firms’ marginal labor cost, \( \frac{W}{P_i} R_i \), and \( R_i = R_i^t \) owing to the perfect competition among the financial intermediaries. Despite the disparity between the one-period bond and the two-period bond determination procedure, the liquidity effect triggered by the wedge of money value in both markets consistently generates a positive term premium in the imperfect information model.
Table 2: Values of Term Premia in the Imperfect Information Model (Basis points)

<table>
<thead>
<tr>
<th>$\gamma = 1$</th>
<th>$\psi = 1$</th>
<th>$s(t)$</th>
<th>$s(t-1)$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>i.i.d.</td>
<td>1</td>
<td>1.85</td>
<td>1.83</td>
<td>1.81</td>
<td>1.83</td>
<td>1.81</td>
<td>1.83</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.10</td>
<td>0.83</td>
<td>2.16</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr.</td>
<td>3</td>
<td>2.15</td>
<td>0.85</td>
<td>2.20</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\gamma = 1$</th>
<th>$\psi = 0.1$</th>
<th>$s(t)$</th>
<th>$s(t-1)$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>i.i.d.</td>
<td>1</td>
<td>1.85</td>
<td>1.83</td>
<td>1.81</td>
<td>1.83</td>
<td>1.81</td>
<td>1.83</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.22</td>
<td>0.87</td>
<td>2.26</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr.</td>
<td>3</td>
<td>2.26</td>
<td>0.89</td>
<td>2.30</td>
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</table>

<table>
<thead>
<tr>
<th>$\gamma = 1$</th>
<th>$\psi = 8$</th>
<th>$s(t)$</th>
<th>$s(t-1)$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>i.i.d.</td>
<td>1</td>
<td>1.85</td>
<td>1.83</td>
<td>1.81</td>
<td>1.83</td>
<td>1.81</td>
<td>1.83</td>
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<tr>
<td></td>
<td>2</td>
<td>2.03</td>
<td>0.81</td>
<td>2.09</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corr.</td>
<td>3</td>
<td>2.07</td>
<td>0.82</td>
<td>2.14</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note 1): $s(t)$ represents the state of money growth realization in this period, $s(t-1)$ the last period.

2): 1 denotes low money growth realization, 2 medium, and 3 high.
4.3.2 The Adjustment Cost Model

As mentioned earlier, the adjustment cost model nests the CIA model and the imperfect information model, depending upon the size of the adjustment cost parameters, $\phi$. From equations (3-29) and (3-30), we can easily see that when $\phi$ is zero,\footnote{\textit{U}_Q and \textit{U}_M terms vanish.} the FOC’s for the one- and the two-period bonds in the adjustment cost model are exactly the same as those in the CIA case. Table 3 and Table 4 show the economic variables’ responses to a money contraction and the term premium signs in the CIA model (when $\phi$ is zero in the adjustment cost model). As expected, they do not show the liquidity effect or the positive term premia.

### Table 3: Responses to a Money Contraction in the CIA Model

\[ ( \phi = 0 \text{ in the Adjustment Cost Model}) \]

<table>
<thead>
<tr>
<th>Vars.</th>
<th>$\psi = 1$</th>
<th>i.i.d. Corr. (Coef. = 0.5)</th>
<th>$\gamma = 1$</th>
<th>$\gamma = 5$</th>
<th>$\gamma = 1$</th>
<th>$\gamma = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1^\dagger$</td>
<td>0.0</td>
<td>0.0</td>
<td>-0.42</td>
<td>-0.43</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_2^\dagger$</td>
<td>0.0</td>
<td>0.0</td>
<td>-0.32</td>
<td>-0.32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_1$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.30</td>
<td>0.31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{INF}_t$</td>
<td>0.0</td>
<td>0.0</td>
<td>-0.41</td>
<td>-0.40</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Refer to the note to Table 1.

### Table 4: Values of Term Premia in the CIA Model

\[ ( \phi = 0 \text{ in the Adjustment Cost Model, Basis points}) \]

<table>
<thead>
<tr>
<th>$\gamma = 1$</th>
<th>$s(t) = 1$</th>
<th>$s(t) = 2$</th>
<th>$s(t) = 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi = 1$ i.i.d.</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Corr.</td>
<td>-0.78</td>
<td>-0.31</td>
<td>-0.79</td>
</tr>
</tbody>
</table>

Note: Refer to the note to Table 2.

When the $\phi$ is increased to 15, the adjustment cost model reveals a meaningful liquidity effect. These results are shown in Table 5. Although the magnitudes are
slightly different from those in Table 1 for the imperfect information model case, all
the signs are consistent with those in Table 1. The most surprising part in this paper is
found in Table 6. Despite the liquidity effect in Table 5, all term premium signs are
negative except for the high money growth state under the assumption of a positive
serial correlation. Moreover, while the average term premium in the imperfect
information model shows a positive sign, it exhibits a negative sign in the adjustment
cost model. This is the critical defect of the adjustment cost model.

To confirm my findings, I can explore the results from a different angle. In the
adjustment cost model, the liquidity effect is driven by the adjustment cost. Figure 5
shows the degrees of the liquidity effect when the adjustment cost parameter varies
from 0 to 20. The degree of the liquidity effect increases as the adjustment cost
parameter gets larger. Note that while in the i.i.d. case the liquidity effect appears with
relatively small parameters such as 1, the same effect in the positive correlation case is
revealed with larger parameters such as 6. Therefore, in order to see the liquidity
effect in $R_{i}^{I}$ and $R_{i}^{II}$ regardless of the money growth correlation, a somewhat larger
parameter such as 7 is required.

The next point is that the adjustment cost of the one-period bonds is always bigger
than that of the two-period bonds. As acknowledged in the previous sub-section, the
one-period bond return is affected by both the bond market and the labor market.
More specifically, the introduction of adjustment cost causes the distortion of marginal
utility in equation (4-6) and the change of labor supply in equation (4-8) in the one-
period bond case, whereas the two-period bond is affected by the adjustment cost only
through equation (4-7). I name this effect the adjustment cost effect. As we have
already seen, the liquidity effect induces a positive term premium, making the
differences between average returns ($R_{i}^{II} - R_{i}^{I}$) more positive. However, the
adjustment cost effect works the opposite way to the liquidity effect. It makes $R_{i}^{II} - R_{i}^{I}$
more negative. The intuition behind this result is that the bigger adjustment cost for
one-period bonds leads investors to require a higher return on one-period bonds to
compensate them for the adjustment cost loss. Hence, unlike with the imperfect
information model in which there is no adjustment cost channel, the adjustment cost
model involves this new distortion.
## Table 5: Responses to a Money Contraction in the Adjustment Cost Model

\( (\phi = 15) \)

<table>
<thead>
<tr>
<th>Vars.</th>
<th>i.i.d. Corr. (Coef. = 0.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \gamma = 1 )</td>
</tr>
<tr>
<td>( R^s_1 )</td>
<td>0.51</td>
</tr>
<tr>
<td>( R^u_1 )</td>
<td>0.25</td>
</tr>
<tr>
<td>( N_1 )</td>
<td>-0.36</td>
</tr>
<tr>
<td>( \text{INF}_1 )</td>
<td>-0.24</td>
</tr>
</tbody>
</table>

## Table 6: Values of Term Premia in the Adjustment Cost Model

\( (\phi = 15, \text{ Basis points}) \)

<table>
<thead>
<tr>
<th>( \gamma = 1 )</th>
<th>( \psi = 1 ) s(t) = 1</th>
<th>( \psi = 1 ) s(t) = 2</th>
<th>( \psi = 1 ) s(t) = 3</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>i.i.d.</td>
<td>-1.07</td>
<td>-1.06</td>
<td>-1.05</td>
<td>-1.06</td>
</tr>
<tr>
<td>Corr.</td>
<td>-55.1</td>
<td>-1.03</td>
<td>52.4</td>
<td>-1.12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \gamma = 1 )</th>
<th>( \psi = 0.1 ) s(t) = 1</th>
<th>( \psi = 0.1 ) s(t) = 2</th>
<th>( \psi = 0.1 ) s(t) = 3</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>i.i.d.</td>
<td>-1.07</td>
<td>-1.06</td>
<td>-1.05</td>
<td>-1.06</td>
</tr>
<tr>
<td>Corr.</td>
<td>-61.2</td>
<td>-1.02</td>
<td>59.1</td>
<td>-1.03</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \gamma = 1 )</th>
<th>( \psi = 8 ) s(t) = 1</th>
<th>( \psi = 8 ) s(t) = 2</th>
<th>( \psi = 8 ) s(t) = 3</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>i.i.d.</td>
<td>-1.08</td>
<td>-1.07</td>
<td>-1.06</td>
<td>-1.07</td>
</tr>
<tr>
<td>Corr.</td>
<td>-52.0</td>
<td>-1.05</td>
<td>49.1</td>
<td>-1.17</td>
</tr>
</tbody>
</table>

Note: Refer to the note to Table 1.
Therefore, bond returns are determined by not only the liquidity effect but also the adjustment cost effect. This implies that the term premium is also directed by the interaction of the liquidity effect and the adjustment cost effect. As a logical consequence, the liquidity effect dominates the adjustment cost effect within a small adjustment cost parameter region, while the adjustment cost effect dominates the liquidity effect within a large parameter region.

Figure 6 shows the changes of average returns. In the i.i.d. case, there is no gap between $R^1_{1t}$ and $R^1_{2t}$ initially. However, as the parameter gets bigger, the gap becomes wider, indicating the dominance of the adjustment cost effect. In the positive correlation case, $R^1_{1t}$ is higher than $R^1_{2t}$ at first, reflecting the absence of the liquidity effect. Note that this gap then gradually narrows in the small parameter region, indicating the faster growth of the liquidity effect. Once the parameter exceeds 4,
however, the gap widens again due to the dominance of the adjustment cost effect. Figure 6 clearly shows the interaction of the liquidity effect and the adjustment cost effect.

The movements of average term premia are shown in Figure 7. Regardless of the money growth correlation, the term premia grow toward a positive direction at first, and then go down. This is another reflection of the competition between the liquidity effect and the adjustment cost effect. When the liquidity effect grows faster, the term premium goes up; when the adjustment cost effect is dominant, it goes down.29

In conclusion, to generate the liquidity effect, somewhat larger parameter values are needed in the adjustment cost model. Ironically, with these larger values, the adjustment cost effect dominates the liquidity effect and results in a negative term premium.

To check the robustness of these results, I tried different utility functions and put the adjustment cost in a budget constraint and a CIA constraint instead of a utility function. These different model settings still show a negative term premium.

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29 Note that the term premium in the i.i.d. case shows a positive sign within a small parameter region. However, in the positive correlation case, the term premium does not show a positive sign within that region due to the absence of the liquidity effect.
Figure 6: Changes of Average Returns

Figure 7: Changes of Average Term Premia (Basis Points)
5 Conclusion

The results of this study provide us with several interesting implications. First of all, I confirm that despite the strong assumption of one-period-ahead decision making, the imperfect information model guarantees the liquidity effect and the positive term premium seen in the data. The adjustment cost model, on the other hand, under after-shock decision-making with a portfolio adjustment cost captures only the liquidity effect but fails to generate the positive term premium. This paper shows that an adjustment cost that drives the liquidity effect in the adjustment cost model also creates an adjustment cost effect, which works the opposite way to the liquidity effect. To generate the liquidity effect, a large adjustment cost is needed; with this big cost, however, the adjustment cost effect dominates the liquidity effect, leading to a negative term premium.

Based upon these findings, I conclude that the liquidity effect does not always guarantee the positive term premium and the sign of term premium depends on the driving force to generate the liquidity effect. Consequently, we can use both models to study the liquidity effect. Caution is, however, necessary in using the adjustment cost model in term structure studies. Further studies should be directed toward inventing another modeling strategy to guarantee the liquidity effect and the positive term premium without the strong assumption of one-period-ahead decision-making. Such studies will help to make our monetary models much richer.
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<Abstract in Korean>

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